Oxford Revise | Edexcel A Level Maths | Answers

- Method (**M**) marks are awarded for showing you know a method and have attempted to apply it.
- Accuracy (**A**) marks should only be awarded if the relevant M marks have been awarded.
- Unconditional accuracy (**B**) marks are awarded independently of M marks. They do not rely on method.
- The abbreviation **o.e.** means 'or equivalent (and appropriate)'.

Please note that:

- efficient use of advanced calculators is expected
- inexact numerical answers should be given to three significant figures unless the question states otherwise; values from statistical tables should be quoted in full
- when a value of g is required, it is taken as $g = 9.8\ \text{m s}^{-2}$ unless stated otherwise in the question.

Chapter 22 Integration

Question	Answer	Extra information	Marks
22.1	$\int (x^2 - x - 2)\mathrm{d}x = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + c$	Expanding brackets and attempting to integrate At least two terms correct All terms correct, including '+ c'	M1 A1 A1
	Total		**3 marks**
22.2	$\int \left(x^{\frac{3}{2}} - \frac{x^{-\frac{1}{3}}}{3} \right) \mathrm{d}x = \frac{2}{5}x^{\frac{5}{2}} - \frac{1}{2}x^{\frac{2}{3}} + c$	Attempting to integrate At least one term completely correct All terms correct, including '+ c'	M1 A1 A1
	Total		**3 marks**

Question	Answer	Extra information	Marks
22.3 (a)	$\int e^{6x}dx = \frac{1}{6}e^{6x} + c$	ke^{6x} Must include '+ c'	M1 A1
22.3 (b)	$\left[\frac{1}{6}e^{6x}\right]_0^k = \frac{1}{6}\left(e^{6k} - e^0\right)$ $\Rightarrow \frac{1}{6}\left(e^{6k} - 1\right) = \frac{21}{2}$ $e^{6k} = 64$ $\Rightarrow k = \frac{1}{6}\ln 64$ $= \frac{1}{6} \times \ln\left(2^6\right)$ $= \ln 2$	Applying limits and subtracting Setting up equation in k and attempting to solve Correct value of k	M1 M1 A1
	Total		**5 marks**
22.4 (a)	$\int k\cos 3x dx = \frac{k}{3}\sin 3x + c$	$k\sin 3x$ Must include '+ c'	M1 A1

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Question	Answer	Extra information	Marks
22.4 (b)	$\left[\frac{k}{3}\sin 3x\right]_0^{\frac{\pi}{12}} = \frac{k}{3}\sin\left(\frac{\pi}{4}\right) - \frac{k}{3}\sin 0$	Applying limits and subtracting	M1
	$\Rightarrow \frac{k}{3}\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{3}$	Setting up equation in k and attempting to solve	M1
	$k = \frac{\sqrt{2}}{\frac{\sqrt{2}}{2}} = 2$	Correct value of k	A1
	Total		**5 marks**
22.5	$\int \sec^2 7x\,dx = \frac{1}{7}\tan 7x + c$	$k\tan 7x$ Must include '+ c'	M1A1 A1
	Total		**3 marks**
22.6	$\int \sec 4x \tan 4x\,dx = \frac{1}{4}\sec 4x + c$	$k\sec 4x$ Must include '+ c'	M1A1 A1
	Total		**3 marks**
22.7	$\int \tan^2 2x\,dx = \int\left(\sec^2 2x - 1\right)dx$ $= \frac{1}{2}\tan 2x - x + c$	Use of trigonometric identity $k\tan 2x - x$ Must include '+ c'	M1 M1 A1
	Total		**3 marks**

Question	Answer	Extra information	Marks
22.8 (a)	$\int 3\cos^2 x\,dx = \frac{3}{2}\int(\cos 2x + 1)\,dx$	Use of trigonometric identity	M1
	$= \frac{3}{4}\sin 2x + \frac{3}{2}x + c$	$k\sin 2x + lx$	M1
		Must include ‘+ c’	A1
22.8 (b)	$\left[\frac{3}{4}\sin 2x + \frac{3}{2}x\right]_0^{\pi} = \left(\frac{3}{4}\sin 2\pi + \frac{3\pi}{2}\right) - \left(\frac{3}{4}\sin 0 + 0\right)$	Applying limits to their (a) and subtracting	M1A1
	$= \frac{3\pi}{2}$	Correct answer	A1
	Total		**6 marks**
22.9	$f(x) = x^3 - x^2 - \frac{1}{x} + c$	Attempting to integrate	M1
		With or without ‘+ c’	A1
	$2 = 1^3 - 1^2 - 1 + c \Rightarrow c = 3$	Substituting into their $f(x)$ to find c	M1
	Hence $y = x^3 - x^2 - \frac{1}{x} + 3$	Correct equation	A1
	Total		**4 marks**
22.10	$\int_4^8 \frac{x^2 - 3x + 1}{x}\,dx = \int_4^8\left(x - 3 + \frac{1}{x}\right)dx$	Splitting the fraction	M1
	$= \left[\frac{1}{2}x^2 - 3x + \ln x\right]_4^8$	Integrating	M1A1
	$= (32 - 24 + \ln 8) - (8 - 12 + \ln 4)$	Substituting	M1
	$= 12 + \ln 2$	Correct answer	A1

Question	Answer	Extra information	Marks
	Total		**5 marks**
22.11	$\int 2\sin 2x\,dx = -\cos 2x + c$ $2 = -\cos 2\pi + c \Rightarrow c = 3$ Hence $y = -\cos 2x + 3$	Use of $\sin 2x$ $k\cos 2x$ Must include '+ c' Substituting into their integral to find c Correct equation	M1 A1 A1 M1 A1
	Total		**5 marks**
22.12 (a)	$v = t^2 - 5t + c$ $7 = 0^2 - 5 \times 0 + c \Rightarrow c = 7$ Hence $v = t^2 - 5t + 7$	Integrating Must include '+ c'. Substituting to find c Correct expression	M1 A1 M1 A1
22.12 (b)	$8 = t^2 - 5t + 7 \Rightarrow t^2 - 5t - 1 = 0$ Hence $t = \dfrac{5 \pm \sqrt{29}}{2}$ The particle is travelling at 8 m s^{-1} when $t = \dfrac{5 + \sqrt{29}}{2}$ (s)	Use of their (a) to form three term quadratic Negative solution must be rejected Allow decimal (5.2 or better)	M1 A1 A1
	Total		**7 marks**
22.13 (a)	$\int_2^5 x^3\,dx$	Correct integral	B1

Question	Answer	Extra information	Marks
22.13 (b)	$\int_2^5 x^3 \mathrm{d}x = \left[\frac{1}{4}x^4\right]_2^5$ $= \left(\frac{1}{4}\times 5^4\right) - \left(\frac{1}{4}\times 2^4\right)$ $= \frac{609}{4}$	Integrating and attempting to evaluate Correct answer	M1 A1
	Total		**3 marks**
22.14	$\lim_{\delta x\to 0}\sum_{x=1}^{8}\sqrt[3]{x}\,\delta x = \int_1^8 \sqrt[3]{x}\ \mathrm{d}x$ $= \left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^8$ $= \left(\frac{3}{4}\times 8^{\frac{4}{3}}\right) - \left(\frac{3}{4}\times 1^{\frac{4}{3}}\right)$ $= \frac{45}{4}$	Converting to integral Attempting to integrate and evaluating Correct answer	B1 M1 A1
	Total		**3 marks**
22.15 (a)	$\int\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)\left(x^{\frac{1}{2}} + 2\right)\mathrm{d}x = \int\left(x + 2x^{\frac{1}{2}} - 1 - 2x^{-\frac{1}{2}}\right)\mathrm{d}x$ $= \frac{1}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} - x - 4x^{\frac{1}{2}} + c$	Converting to index form and expanding Attempting to integrate	M1A1 M1A1

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Question	Answer	Extra information	Marks
22.15 (b)	$\left[\frac{1}{2}x^2+\frac{4}{3}x^{\frac{3}{2}}-x-4x^{\frac{1}{2}}\right]_1^9$	Substituting into their (a)	M1
	$=\left(\frac{81}{2}+36-9-12\right)-\left(\frac{1}{2}+\frac{4}{3}-1-4\right)$		
	$=\frac{176}{3}$	Correct answer	A1
	Total		**6 marks**
22.16	$(x + 2)(x^2 – 3x – 4) = (x + 2)(x – 4)(x + 1)$	Factorising to find roots	M1
	Roots at –2, 4 and –1	All roots correct	A1
	Area between –2 and –1 = $\int_{-2}^{-1}\left(x^3-x^2-10x-8\right)\mathrm{d}x$		
	$\left[\frac{1}{4}x^4-\frac{1}{3}x^3-5x^2-8x\right]_{-2}^{-1}=\frac{11}{12}$	One correct integral One correct area	M1 A1
	Area between –1 and 4 = $\int_{-1}^{4}\left(x^3-x^2-10x-8\right)\mathrm{d}x$		
	$\left[\frac{1}{4}x^4-\frac{1}{3}x^3-5x^2-8x\right]_{-1}^{4}=-\frac{875}{12}$		
	Hence area = $\frac{11}{12}+\frac{875}{12}$	Adding two areas, one of which is negative	M1
	$=\frac{886}{12}=\frac{443}{6}$	Correct answer	A1

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Question	Answer	Extra information	Marks
	Total		**6 marks**
22.17	Area = $\int_0^{\pi} \sin t \times (6t+1)\ \mathrm{d}t = \int_0^{\pi} (6t\sin t + \sin t)\ \mathrm{d}t$ $= \left[-6t\cos t + 6\sin t - \cos t\right]_0^{\pi}$ $= 19.849\ldots + 1$ $= 20.849$ Hence area = 20.8 units^2	Use of formula for parametric integration Attempting to integrate Evaluating Correct answer	M1A1 M1A1 M1 A1
	Total		**6 marks**
22.18	$x^2 - 5x - 4 = 8 - x \Rightarrow x^2 - 4x - 12 = 0$ Hence $x = 6$ and -2 $\int_{-2}^{6} (8-x)\mathrm{d}x = \left[8x - \frac{1}{2}x^2\right]_{-2}^{6}$ $= 48$ $\int_{-2}^{6} \left(x^2 - 5x - 4\right)\mathrm{d}x = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 - 4x\right]_{-2}^{6}$ $= -\frac{112}{3}$ Area = $48 - \left(-\frac{112}{3}\right)$ $= \frac{256}{3}$	Attempting to find limits Correct limits One correct integral Subtracting Correct answer	M1 A1 M1A1 M1 A1

Question	Answer	Extra information	Marks
	Total		**6 marks**
22.19	$x^2 - 4x + 2 = 2 + 3x - x^2 \Rightarrow 2x^2 - 7x = 0$ Hence $x = 0$ and 3.5 $\int_0^{3.5}\left(2+3x-x^2\right)\mathrm{d}x = \left[2x+\frac{3}{2}x^2-\frac{1}{3}x^3\right]_0^{3.5}$ $= \frac{133}{12}$ $\int_0^{3.5}\left(x^2-4x+2\right)\mathrm{d}x = \left[\frac{1}{3}x^3-2x^2+2x\right]_0^{3.5}$ $= -\frac{77}{24}$ Area $= \frac{133}{12} - \left(-\frac{77}{24}\right)$ $= \frac{343}{24}$	Attempting to find limits Correct limits One correct integral Subtracting Correct answer	M1 A1 M1A1 M1 A1
	Total		**6 marks**
22.20 (a)	$AB^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \cos 67°$ $= 124.635\ldots$ Hence $AB = 11.164\ldots$ $= 11.2$ cm (3 s.f.)	Use of cosine rule Taking the square root Correct answer	M1 M1 A1

Question	Answer	Extra information	Marks
22.20 (b)	$\frac{\sin CAB}{11}=\frac{\sin 67°}{11.164...}$ $\sin CAB=\frac{11\sin 67°}{11.164...}$ $CAB=65.1°$ (1 d.p.)	Use of sine rule with their (a) Rearranging and solving Correct answer	M1 M1 A1
22.20 (c)	$\frac{1}{2}\times 9\times 11\times \sin 67°$ $= 45.564\ldots$ $= 45.6\text{ cm}^2$ (3 s.f.)	Use of area of triangle formula Correct answer	M1 A1
	Total		**8 marks**
22.21 (a)	$\frac{ar^5}{ar^2}=\frac{\frac{1}{24}}{\frac{1}{3}}\Rightarrow r^3=\frac{1}{8}$ $r=\frac{1}{2}$ $a\left(\frac{1}{2}\right)^2=\frac{1}{3}$ $a=\frac{4}{3}$	Attempting to find r^3 Substituting to find a Correct answer	M1A1 M1 A1

Question	Answer	Extra information	Marks
22.21 (b)	$\frac{4}{3}\times\left(\frac{1}{2}\right)^7$ $=\frac{1}{96}$	Substituting values of *a* and *r* and calculating Correct answer	M1 A1
22.21 (c)	$S_7=\frac{\frac{4}{3}\left[1-\left(\frac{1}{2}\right)^7\right]}{1-\frac{1}{2}}$ $=\frac{127}{48}$	Use of summation formula with $n = 7$ Correct answer	M1 A1
22.21 (d)	$S_\infty=\frac{\frac{4}{3}}{1-\frac{1}{2}}$ $=\frac{8}{3}$	Use of sum to infinity formula Correct answer	M1 A1
	Total		**10 marks**

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Question	Answer	Extra information	Marks
22.22	Midpoint of $AC = (4, 3.5)$ The centre lies on the line $y = 3.5$ Midpoint of $AB = (1.5, 2.5)$ [or midpoint of $BC = (1.5, 6)$] Gradient of $AB = -1$ Gradient of perpendicular bisector of $AB = 1$ Equation of bisector is $y - 2.5 = x - 1.5$ or $y = x + 1$ When $y = 3.5$: $3.5 = x + 1$ $x = 2.5$ Centre = $(2.5, 3.5)$ Radius = $\sqrt{14.5}$ Equation of circle is $(x - 2.5)^2 + (y - 3.5)^2 = 14.5$	Finding a midpoint using any appropriate method Centre lies on the perpendicular bisector of AC Finding one other midpoint Finding a perpendicular bisector of one of the lines, then setting $y = 3.5$ and solving for x Correct centre Using the centre and one point on circumference to find radius Substituting information into the equation for a circle	M1 M1 M1 M1 A1 A1 M1A1
	Total		**8 marks**