Oxford Revise | Edexcel A Level Maths | Answers

- Method (**M**) marks are awarded for showing you know a method and have attempted to apply it.
- Accuracy (**A**) marks should only be awarded if the relevant M marks have been awarded.
- Unconditional accuracy (**B**) marks are awarded independently of M marks. They do not rely on method.
- The abbreviation **o.e.** means 'or equivalent (and appropriate)'.

Please note that:

- efficient use advanced calculators is expected
- inexact numerical answers should be given to three significant figures unless the question states otherwise; values from statistical tables should be quoted in full
- when a value of g is required, it is taken as $g = 9.8\ \text{m s}^{-2}$ unless stated otherwise in the question.

Chapter 21 Differentiation methods

Question	Answer	Extra information	Marks
21.1 (a)	$f'(x) = \dfrac{2x}{x^2 - 3}$	Attempting to differentiate using chain rule Correct numerator and denominator	M1 A1A1
21.1 (b)	Since $x > \sqrt{3}$, the denominator is always positive.	Correct answer and conclusion	B1
	Total		**4 marks**
21.2 (a)	$f'(x) = 8x \cos(4x^2 - 3)$	Attempting to differentiate using chain rule For $8x$ Fully correct derivative	M1 A1 A1

Question	Answer	Extra information	Marks
21.2 (b)	$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$ $4x^2 - 3 = \frac{\pi}{2}$ $\Rightarrow x = \sqrt{\frac{\pi+6}{8}}$ $y = \sin\left[4\left(\sqrt{\frac{\pi+6}{8}}\right)^2 - 3\right]$ $= \sin\left[4\left(\frac{\pi+6}{8}\right) - 3\right]$ $= \sin\frac{\pi}{2}$ $= 1$ So P has coordinates $\left(\sqrt{\frac{\pi+6}{8}}, 1\right)$	Finding x for maximum point Attempting to solve Correct x-coordinate Correct y-coordinate	M1 M1 A1 A1
	Total		**7 marks**
21.3 (a)	$f'(x) = (6x + 1)e^{3x^2+x}$	Attempting to differentiate using chain rule For $(6x + 1)$ Fully correct derivative	M1 A1 A1

Question	Answer	Extra information	Marks
21.3 (b)	$6x + 1 = 0$ $x = -\frac{1}{6}$	Attempting to solve $f'(x) = 0$ Correct x-coordinate	M1 A1
21.3 (c)	$f''(x) = 6e^{3x^2+x} + (6x+1)^2 e^{3x^2+x}$	Use of product rule A1 for $6e^{3x^2+x}$ A1 for $(6x + 1)^2$ A1 for e^{3x^2+x}	M1 A1 A1 A1
21.3 (d)	$6e^{3\times\left(-\frac{1}{6}\right)^2-\frac{1}{6}} + \left[6\times\left(-\frac{1}{6}\right)+1\right]^2 e^{3\times\left(-\frac{1}{6}\right)^2-\frac{1}{6}} = 5.520...$ Since $f''\left(-\frac{1}{6}\right) > 0$, so the stationary point is a minimum.	Correct method and conclusion	B1
	Total		**10 marks**
21.4 (a)	$f'(x) = 2x\cos x - x^2 \sin x$	Use of product rule One term correct Both terms correct	M1 A1 A1
21.4 (b)	$f'(x) = 0 \Rightarrow x(2\cos x - x\sin x) = 0$ so $x = 0$ is a solution to $f'(x) = 0$	For factorising x Correct conclusion	M1 A1
21.4 (c)	$f''(x) = 2\cos x - 2x\sin x - (2x\sin x - x^2\cos x)$ $= 2\cos x - 4x\sin x + x^2\cos x$	Use of product rule For unsimplified derivative Fully simplified expression	M1 A1 A1

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Question	Answer	Extra information	Marks
21.4 (d)	$f''(0) = 2$ Since $f''(0) > 0$, the stationary point is a minimum.	Correct conclusion	B1
	Total		**9 marks**
21.5 (a)	$f'(x) = 3e^x \cos 3x + e^x \sin 3x$ $f'(x) = 0 \Rightarrow 3\cos 3x + \sin 3x = 0$ Hence $\tan 3x = -3$ $3x = 1.8925\ldots$ $x = 0.6308\ldots$ $= 0.63$ $y = e^{0.63\ldots} \times \sin(3 \times 0.63\ldots)$ $= 1.7827\ldots$ $= 1.78$	Use of product rule Correct derivative Factorising e^x and equating the resulting expression to 0 For any correct solution to $\tan 3x = -3$ Correct x-coordinate Correct y-coordinate	M1 A1 M1 A1 A1 A1
21.5 (b)	$f''(x)$ $= 3e^x \cos 3x + 3e^x(-3\sin 3x)$ $+ e^x \sin 3x + 3e^x \cos 3x$ $= 3e^x \cos 3x - 9e^x \sin 3x + e^x \sin 3x + 3e^x \cos 3x$ $= 6e^x \cos 3x - 8e^x \sin 3x$	Use of product rule Correct derivative of $3e^x \cos 3x$ Correct derivative of $e^x \sin 3x$ Correct fully simplified answer	M1 A1 A1 A1
21.5 (c)	$f'(0.63) = -17.827\ldots < 0$ therefore it is a maximum.		B1
	Total		**11 marks**

Question	Answer	Extra information	Marks
21.6 (a)	$f'(x) = \dfrac{-x\sin x - \cos x}{x^2}$	Use of quotient rule Each of three terms correct	M1 A1A1A1
21.6 (b)	$f'\left(\dfrac{\pi}{3}\right) = \dfrac{-\left(\dfrac{\pi}{3}\right)\sin\left(\dfrac{\pi}{3}\right) - \cos\left(\dfrac{\pi}{3}\right)}{\left(\dfrac{\pi}{3}\right)^2} = -1.28$	Substituting x-value into their (a)	M1
	$f\left(\dfrac{\pi}{3}\right) = \dfrac{\cos\left(\dfrac{\pi}{3}\right)}{\left(\dfrac{\pi}{3}\right)} = 0.48$	Correct y-coordinate	A1
	Hence $y - 0.48 = -1.28\left(x - \dfrac{\pi}{3}\right)$	Use of any correct formula with their f and f′	M1
	$\Rightarrow y = -1.28x + 1.82$	Correct line	A1
	Total		**8 marks**
21.7 (a)	$\dfrac{dy}{dx} = \dfrac{2e^{2x}\sin x - e^{2x}\cos x}{(\sin x)^2}$	Use of quotient rule Each of three terms correct	M1 A1A1A1

Question	Answer	Extra information	Marks
21.7 (b)	When $x = \frac{\pi}{4}$ $\frac{dy}{dx} = \frac{2e^{2\times\frac{\pi}{4}}\sin\left(\frac{\pi}{4}\right) - e^{2\times\frac{\pi}{4}}\cos\left(\frac{\pi}{4}\right)}{\left[\sin\left(\frac{\pi}{4}\right)\right]^2}$	Substituting $x = \frac{\pi}{4}$	M1
	$= \frac{2e^{\frac{\pi}{2}}\frac{\sqrt{2}}{2} - e^{\frac{\pi}{2}}\frac{\sqrt{2}}{2}}{\left(\frac{\sqrt{2}}{2}\right)^2}$	Substituting trig values and simplifying	M1
	$= \frac{e^{\frac{\pi}{2}}\frac{\sqrt{2}}{2}}{\frac{1}{2}}$		
	$= \sqrt{2}e^{\frac{\pi}{2}}$	Correct gradient. Must be exact.	A1
	Total		**7 marks**
21.8 (a)	$u = e^{4x+1} \Rightarrow u' = 4e^{4x+1}$	Attempting to find u'	M1
	$v = \sin(x^2) \Rightarrow v' = 2x\cos(x^2)$	Attempting to find v'	M1
	Hence $f'(x) = 4e^{4x+1}\sin(x^2) + 2x\,e^{4x+1}\cos(x^2)$	Use of product rule	M1
	$= 2e^{4x+1}[2\sin(x^2) + x\cos(x^2)]$	Completely correct solution	A1

Question	Answer	Extra information	Marks
21.8 (b)	In the given domain, $2\sin(x^2) + x\cos(x^2)$ is always positive (square values of x between 0 and 1 and then the sine or cosine gives a positive output). In the given domain, e^{4x+1} is always positive (exponentials of this form always return positive outputs). Hence $f'(x) > 0$ for the given domain.	Sufficient detail must be given to make it clear that the student has considered the sign of both parts of the product Correct conclusion	B1 B1
	Total		**6 marks**
21.9 (a)	$\frac{dx}{dy} = \ln y + \frac{y}{y}$ $= \ln y + 1$	Use of product rule Each term. Must be simplified.	M1 A1A1
21.9 (b)	$\frac{dy}{dx} = \frac{1}{\ln y + 1}$ When $y = 2e$: $\frac{dy}{dx} = \frac{1}{\ln(2e) + 1}$ $= \frac{1}{\ln 2 + 2}$ $(= 0.3713\ldots)$	Use of inverse Correct answer. Accept decimal.	M1 A1
	Total		**5 marks**

Question	Answer	Extra information	Marks
21.10 (a)	$8x + 6y\frac{dy}{dx} - 5x\frac{dy}{dx} - 5y = 0$ Hence $\frac{dy}{dx} = \frac{5y - 8x}{6y - 5x}$ (o.e.)	Use of implicit differentiation on $6y^2$ or $5xy$ Use of implicit differentiation on both $6y^2$ and $5xy$ Correct equation Attempting to rearrange Fully correct derivative	M1 M1 A1 M1 A1
21.10 (b)	At (2, 1): $\frac{dy}{dx} = \frac{5-16}{6-10} = \frac{11}{4}$ $m_N = -\frac{4}{11}$ Hence $y - 1 = -\frac{4}{11}(x - 2)$ $4x + 11y - 19 = 0$	Using their derivative perpendicular gradient Correct gradient Use of any correct formula with their perpendicular gradient Correct line in correct form	M1 A1 M1 A1
	Total		**9 marks**
21.11	$e^x + e^y\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$ Hence $\frac{dy}{dx} = \frac{3y - e^x}{e^y - 3x}$ (o.e.)	Use of implicit differentiation on e^y or $3xy$ Use of implicit differentiation on both e^y and $3xy$ Correct equation Attempting to rearrange Fully correct derivative	M1 M1 A1 M1 A1
	Total		**5 marks**

Question	Answer	Extra information	Marks
21.12	$\frac{dx}{dt} = 2t$; $\frac{dy}{dt} = 3t^2$	Use of rule for parametric differentiation	M1
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		
	$= \frac{3t^2}{2t}$		
	$= \frac{3}{2}t$	Correct $\frac{dy}{dx}$	A1
	Hence $\frac{dy}{dx} = 3$	Evaluating at $t = 2$	M1
	$m_N = -\frac{1}{3}$	Correct perpendicular gradient	A1
	$x = 4$, $y = 8$	Both x and y coordinates correct	B1
	Hence $y - 8 = -\frac{1}{3}(x - 4)$	Use of any correct formula with their perpendicular gradient	M1
	$3y + x - 28 = 0$ (o.e.)	Correct line	A1
	Total		**7 marks**

Question	Answer	Extra information	Marks
21.13 (a)	$\frac{dx}{dt}=2\cos 2t$ $\frac{dy}{dt}=-2\sin 2t$ $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ $=\frac{-2\sin 2t}{2\cos 2t}$ $=-\tan 2t$	Correct derivative Correct derivative Use of rule for parametric differentiation Correct derivative	B1 B1 M1 A1
21.13 (b)	$-\tan\left(\frac{\pi}{4}\right)=-1$ $x=\frac{\sqrt{2}}{2}, y=\frac{\sqrt{2}}{2}$ Hence $y-\frac{\sqrt{2}}{2}=-\left(x-\frac{\sqrt{2}}{2}\right)$ $2x+2y-2\sqrt{2}=0$ (o.e.)	Evaluating derivative at $t=\frac{\pi}{8}$ Both x and y coordinates correct Use of any correct formula with their m Correct line	M1 B1 M1 A1
	Total		**8 marks**
21.14 (a)	$\frac{dx}{dt}=3-2t$; $\frac{dy}{dt}=3t^2-12$ $\frac{dy}{dx}=\frac{3t^2-12}{3-2t}$	Both derivatives correct Use of rule for parametric differentiation Correct derivative	B1 M1 A1

Question	Answer	Extra information	Marks
21.14 (b)	$3t^2 - 12 = 0 \Rightarrow t = \pm 2$ When $t = 2$: $x = 2$ and $y = -16$ (hence P) $3 - 2t = 0 \Rightarrow t = \frac{3}{2}$ When $t = \frac{3}{2}$: $x = \frac{9}{4}$ (and $y = -\frac{117}{8}$) (hence Q) Area = 22.5 × 160 = 3600 (m^2)	Setting the numerator of $\frac{dy}{dx} = 0$ and attempting to solve Correct y-coordinate Setting the denominator of $\frac{dy}{dx} = 0$ and attempting to solve Correct x-coordinate Calculating area using their $x \times 10$ and their $y \times 10$ Correct area	M1 A1 M1 A1 M1 A1
	Total		**9 marks**
21.15 (a)	Let the side length of the cube = x $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ $= 3x^2 \times 5$ $= 15x^2$ Hence, when $x = 23$, $\frac{dV}{dt} = 7935\,\text{cm}^3$ per second	Attempting to find derivative of V with respect to x Correct derivative Use of formula for connected rate of change Correct result	M1 A1 M1 A1

Question	Answer	Extra information	Marks
21.15 (b)	$A = 6x^2 \Rightarrow \frac{dA}{dx} = 12x$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= 12x \times 5$ $= 60x$ Hence, when $x = 18$, $\frac{dA}{dt} = 1080\ \text{cm}^2$ per second	Attempting to find derivative of A with respect to x Correct derivative Use of formula for connected rate of change Correct result	M1 A1 M1 A1
	Total		**8 marks**
21.16 (a)	Let the radius of the sphere = r $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$ $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $= 4\pi r^2 \times -0.5$ $= -2\pi r^2$ Hence, when $r = 8$, $\frac{dV}{dt} = -128\pi\ \text{cm}^3$ per second	Attempting to find derivative of V with respect to r Correct derivative Use of formula for connected rate of change Correct result	M1 A1 M1 A1

Question	Answer	Extra information	Marks
21.16 (b)	$A = 4\pi r^2 \Rightarrow \dfrac{dA}{dr} = 8\pi r$	Attempting to find derivative of A with respect to r	M1
		Correct derivative	A1
	$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt}$ $= 8\pi r \times -0.5$ $= -4\pi r$	Use of formula for connected rate of change	M1
	Hence, when $r = 10$, $\dfrac{dA}{dt} = -40\pi\ \text{cm}^2$ per second	Correct result	A1
	Total		**8 marks**
21.17	$f(x) = (x + 4)^2 + 4$	Attempting to complete the square	M1
		Correct expression	A1
	Since $(x + 4)^2 \geq 0$ for all x, $f(x) \geq 4$ for all x	Correct reasoning	A1
	Total		**3 marks**
21.18	$e^x + 6 - 5e^{-x} = 0$	Multiplying by e^x	M1
	$\Rightarrow e^{2x} + 6e^x - 5 = 0$	Quadratic equation	A1
	$e^x = -3 \pm \sqrt{14}$	Solving for e^x using calculator or quadratic formula	M1
	Hence $x = \ln\left(-3 + \sqrt{14}\right)$	Correct x	A1
	Total		**4 marks**

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