

Oxford Revise | AQA GCSE Maths Higher | Answers

Chapter 27 Vectors

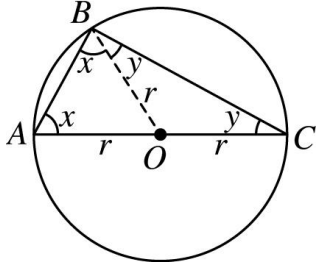
Question	Answer	Extra information	Marks
27.1 (a)		<p>Correct vector drawn</p> <p>Arrow pointing in the correct direction</p>	<p>1</p> <p>1</p>
27.1 (b)	$2\mathbf{b} + 3\mathbf{a} = 2\begin{pmatrix} 2 \\ -1 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 9 \end{pmatrix}$ $= \begin{pmatrix} 7 \\ 7 \end{pmatrix}$	<p>2b and 3a both correct</p> <p>Correct vector</p>	<p>1</p> <p>1</p>
27.2 (a)	$\overrightarrow{OT} = \overrightarrow{QO} = -\mathbf{a}$		1
27.2 (b)	$\overrightarrow{PQ} = -\mathbf{b} + \mathbf{a}$		1

Question	Answer	Extra information	Marks
27.2 (c)	$\overrightarrow{OU} = \mathbf{b} - \mathbf{a}$		1
27.2 (d)	$\overrightarrow{UQ} = \overrightarrow{UO} + \overrightarrow{OQ}$ $= \mathbf{a} - \mathbf{b} + \mathbf{a}$ $= 2\mathbf{a} - \mathbf{b}$		1
27.3	<p>Let \mathbf{q} be the vector $\mathbf{q} = \begin{pmatrix} x \\ y \end{pmatrix}$</p> $2\mathbf{q} - 5\mathbf{p} = 2\begin{pmatrix} x \\ y \end{pmatrix} - 5\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ $= \begin{pmatrix} 2x - 20 \\ 2y + 5 \end{pmatrix}$ <p>Therefore $2x - 20 = -26 \Rightarrow x = -3$ and $2y + 5 = 15 \Rightarrow y = 5$</p> $\mathbf{q} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$	<p>Letting $\mathbf{q} = \begin{pmatrix} x \\ y \end{pmatrix}$ and writing</p> $2\mathbf{q} - 5\mathbf{p} = 2\begin{pmatrix} x \\ y \end{pmatrix} - 5\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ <p>Equating the vector components and attempting to solve for x and y. Correct answer</p>	<p>1</p> <p>1</p> <p>1</p>
27.4	$\overrightarrow{OC} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$	<p>C correctly plotted Correct column vector</p>	<p>1</p> <p>1</p>

Question	Answer	Extra information	Marks
27.5	$\overline{AB} = \overline{AO} + \overline{OB} = \mathbf{b} - \mathbf{a}$ $\overline{MN} = \overline{MA} + \overline{AN} = \frac{1}{2}\overline{OA} + \frac{4}{5}\overline{AB}$ $= \frac{1}{2}\mathbf{a} + \frac{4}{5}(\mathbf{b} - \mathbf{a})$ $= \frac{4}{5}\mathbf{b} - \frac{3}{10}\mathbf{a}$	1 mark for each line	4
27.6	$\overline{PQ} = \overline{PO} + \overline{OQ} = \mathbf{q} - \mathbf{p}$ $PQ : QR = 2 : 3, \overline{PR} = \frac{5}{2}\overline{PQ} = \frac{5}{2}(\mathbf{q} - \mathbf{p})$ $\overline{OR} = \overline{OP} + \overline{PR} = \mathbf{p} + \frac{5}{2}(\mathbf{q} - \mathbf{p}) = \frac{5}{2}\mathbf{q} - \frac{3}{2}\mathbf{p}$ $= \frac{1}{2}(5\mathbf{q} - 3\mathbf{p})$ <p>This is a multiple of $5\mathbf{q} - 3\mathbf{p}$, so it is parallel</p>	1 mark for each line	5

Question	Answer	Extra information	Marks
27.7	$\overrightarrow{OP} = \frac{3}{4}\mathbf{a}, \overrightarrow{PA} = \frac{1}{4}\mathbf{a}, \overrightarrow{OQ} = k\mathbf{b}$ <p>where k is a scalar constant.</p> $\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -\mathbf{a} + k\mathbf{b}$ $\overrightarrow{PM} = \overrightarrow{PA} + \overrightarrow{AM} = \overrightarrow{PA} + \frac{1}{2}\overrightarrow{AQ}$ $= \frac{1}{4}\mathbf{a} + \frac{1}{2}(-\mathbf{a} + k\mathbf{b}) = -\frac{1}{4}\mathbf{a} + \frac{1}{2}k\mathbf{b}$ $\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB} = -\frac{3}{4}\mathbf{a} + \mathbf{b}$ $= 3\overrightarrow{PM}$ <p>So $1 = 3 \times \frac{1}{2}k$</p> $k = \frac{2}{3}$ <p>Substituting into \overrightarrow{OQ}:</p> $\overrightarrow{OQ} = \frac{2}{3}\mathbf{b}$ <p>So $OQ : QB = 2 : 1$</p>	$\overrightarrow{OP} = \frac{3}{4}\mathbf{a} \text{ or } \overrightarrow{PA} = \frac{1}{4}\mathbf{a}$ <p>Attempt to find \overrightarrow{AQ} in terms of \mathbf{a} and \mathbf{b}</p> <p>Use of $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AQ}$</p> <p>Full process to solve for k</p> <p>Final answer</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>

Question	Answer	Extra information	Marks
27.8	$\overline{CB} = 3\mathbf{a}$ $\overline{AC} = \overline{AO} + \overline{OC} = -9\mathbf{a} + \mathbf{c}$ $\overline{AP} = \frac{3}{4}\overline{AC} = -\frac{27}{4}\mathbf{a} + \frac{3}{4}\mathbf{c}$ $\overline{OB} = \overline{OC} + \overline{CB} = 3\mathbf{a} + \mathbf{c}$ $\overline{OP} = \overline{OA} + \overline{AP} = 9\mathbf{a} - \frac{27}{4}\mathbf{a} + \frac{3}{4}\mathbf{c} = \frac{9}{4}\mathbf{a} + \frac{3}{4}\mathbf{c}$ $\overline{OP} = \frac{3}{4}\overline{OB}$ Thus O, P and B are collinear	$\overline{CB} = 3\mathbf{a}$ Attempt to find \overline{AP} in terms of \mathbf{a} and \mathbf{b} Attempt to find \overline{OP} in terms of \mathbf{a} and \mathbf{b} Shows that \overline{OP} and \overline{OB} are multiples, thus collinear	1 1 1 1
27.9 (a)	Convert km/h to m/s $8 \text{ km/h} = \frac{8 \times 1000}{60 \times 60} \text{ m/s}$ $= 2.22\dots \text{ m/s}$ Jordan is faster	Correct calculation for conversion Clear comparison of the two speeds	1 1
27.9 (b)	Distance = 8.5 km $2.5 \text{ m/s} = \frac{2.5 \times 60 \times 60}{1000} = 9 \text{ km/h}$ Combined speed of runners = $8 + 9 = 17 \text{ km/h}$ $8.5 \div 17 = 0.5 \text{ hours}$ The runners will pass each other after 30 minutes	Attempt to combine speeds Dividing total distance by combined speeds Correct answer	1 1 1

Question	Answer	Extra information	Marks
27.9 (c)	Riley's speed is 8 km/h = 4 km per 30 minutes The runners meet 4 km from point A	Attempt to convert 8 km/h to km per your time from part b Correct answer, including units (4 km or 4000 m)	1 1
27.10	<p>Consider an unknown angle (ABC) in a semicircle as shown.</p>  <p>Let $\angle BAC = x$ and let $\angle BCA = y$ Let the radius of the circle be r.</p> <p>$OC = OA = OB = r$ As the base angles of an isosceles triangle are equal, $\angle OBA = x, \angle OBC = y$ So, $\angle ACB = x + y$ Angles in a triangle add to 180°: $x + y + x + y = 180$ $2(x + y) = 180$ $x + y = 90$ Therefore, the angle in a semicircle is a right angle</p>	<p>Clear use of isosceles triangle to identify angle $OCA (= x)$ or angle $OCB (= y)$ $ACB = x + y$ $x + y + x + y = 180$ Fully correct proof</p>	1 1 1 1