Oxford Revise | AQA GCSE Maths Higher | Answers

Chapter 12 Sequences

Question	Answer	Extra information	Marks
12.1 (a)	$35 - 4n < 0$ $35 < 4n$ $8.75 < n$ This is the 9th term $35 - 9(4) = 35 - 36 = -1$		1 1
12.1 (b)	$35 - 4n = -100$ $135 = 4n$ $n = 33.75$ n is not an integer, so -100 is not in the sequence.		1 1
12.2 (a)	$8n + 3 = 51$; $8n = 48$; $n = 48 \div 8 = 6$ The 6th term is 51	Writing the equation Correct answer.	1 1
12.2 (b)	$8n + 3 = 64$; $8n = 61$ 61 is not divisible by 8, so 64 is not in the sequence.	Writing the equation Correct answer.	1 1
12.2 (c)	$8n + 3 > 100$; $8n > 97$; $> 97 \div 8$ (= 12.125) This is the 13th term. The 13th term is $8 \times 13 + 3 = 107$	Writing the inequality 13th term Correct answer	1 1 1
12.3 (a)	When $n = 4$, $n^2 - 30 = 4^2 - 30$ $= 16 - 30 = -14$	Substituting in 4 Correct answer	1 1

ISBN 9781382039833

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12.3 (b)	$n^2 - 30 = 114$, so $n^2 = 144$. Since 144 is a square number, and $n = 12$, this is in the sequence.	Writing the equation Correct answer.	1 1
12.4 (a) (i)	With a Fibonacci sequence, you add together the previous two terms. The sequence begins: $m, n, m + n, m + 2n, 2m + 3n, 3m + 5n, 5m + 8n, \ldots$ The fourth term is $m + 2n$		1
12.4 (a)	The seventh term is $5m + 8n$	Finding the fifth and sixth terms Correct answer.	1 1
12.4 (b)	$m = 3$ The gap between the 1st and 3rd is: $(m + n) - m = n$ so $n = 5$ The 8th term is $8m + 13n = 8 \times 3 + 13 \times 5 = 89$	Method for finding the 8th term Correct answer	1 1
12.5	The nth term is given by $\frac{1}{2} \times \left(\frac{1}{3}\right)^{n-1}$		3
12.6	The sequence begins 5, __ , 11, … Since it is arithmetic, it increases by the same amount each time. In two jumps, it increases by 6, so the term-to-term rule is 'add 3' and the sequence is 5, 8, 11, … This makes the nth term $3n + 2$	Identifying the sequence nth term. 50th & 60th term	1 1 1

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12.7	The sequence $12, 9, 6, 3, \ldots$ has nth term $15 - 3n$ The 50th term is $15 - 3 \times 50 = -135$ and the 60th term is $15 - 3 \times 60 = -165$ The sum of these terms is $(-135) + (-165) = -300$	Finding the nth term Finding the 50th and 60th terms Correct answer.	1 1 1
12.8 (a)	The next term will be $\frac{13}{6}$		1
12.8 (b)	The nth term is given by $\frac{2n+1}{n}$		3
12.8 (c)	$\frac{2\times6+1}{6}\times\frac{2\times9+1}{9}=\frac{13}{6}\times\frac{19}{9}=\frac{247}{54}$		2
12.9	$\begin{array}{lcccc} & 1, & 12, & 27, & 46, \ldots \\ \text{First differences:} & +11 & +15 & +19 & \ldots \\ \text{Second differences:} & +4 & +4 & & \end{array}$ Sequence involves $2n^2$ $\begin{array}{lcccc} & 1 & 12 & 27 & 46 \\ 2n^2 & 2 & 8 & 18 & 32 \\ & -1 & 4 & 9 & 14 \end{array}$ Linear sequence: $-1, 4, 9, 14$ Difference between terms is $+5$ nth term $= 5n - 6$ nth term of quadratic sequence $= 2n^2 + 5n - 6$		1 1 1 1 1 1

Question	Answer	Extra information	Marks
12.10	$-1, \quad -5, \quad -11, \quad -19, \ldots$ First differences: $-4 \quad -6 \quad -8$ Second differences: $-2 \quad -2$ Sequence involves $-n^2$ $\begin{array}{lcccc} & -1 & -5 & -11 & -19 \\ n^2 & -1 & -4 & -9 & -16 \\ & 0 & 1 & 2 & 3 \end{array}$ Linear sequence: $0, 1, 2, 3$ Difference between terms is $+1$ nth term $= n - 1$ nth term of quadratic sequence $= -n^2 + n - 1$		1 1 1 1 1 1
12.11	$n^2 + 2n + 2 = 50 \Rightarrow n^2 + 2n - 48 = 0$ $\Rightarrow (n + 8)(n - 6) = 0$ So, the solutions are $n = -8$ or $n = 6$ Since n is a positive number, $n = 6$ So, the 6th term is 50	Writing the nth term equal to 50 Rearranging to 0 and attempting to solve the quadratic by factorising (or equivalent method of solution) Correct answer	1 1 1

Question	Answer	Extra information	Marks
12.12	$n = 2$: $4 + 2b + c = 13$ $2b + c = 9$ $n = 5$: $25 + 5b + c = 40$ $5b + c = 15$ Form two equations: $2b + c = 9$ (1) $5b + c = 15$ (2) (2) – (1): $5b + c = 15$ $2b + c = 9$ $3b = 6$ $b = 2$ Substitute into (1): $4 + c = 9$ $c = 5$ nth term = $n^2 + 2n + 5$	Method to find an equation in b and c. Finds a pair of simultaneous equations, and an attempt to eliminate b. $b = 2$ $c = 5$ Correct final answer	1 1 1 1 1

Question	Answer	Extra information	Marks
12.13	$n=4 \Rightarrow 16a+b=42$ $n=9 \Rightarrow 81a+b=237$ Subtract the first equation from the second: $65a=195$ $a=3$ Substitute this into either equation to get $b=-6$ So, the nth term is $3n^2-6$ 15th term will be $3\times15^2-6=669$	Method to find an equation in a and b. Finds a pair of simultaneous equations, and an attempt to eliminate b. $a = 3$ and $b = -6$ Substitutes $n = 15$ into formula Correct final answer	1 1 1 1 1
12.14	$\frac{4}{9+\sqrt{y}}=\frac{9-\sqrt{y}}{4}$ $(9+\sqrt{y})(9-\sqrt{y})=16$ $81-y=16$ $y=65$	Sets up correct equation Attempt to expand and solve for y Correct answer	1 1 1
12.15 (a)	$\frac{1}{4},\frac{2}{5},\frac{3}{6}$	Substitutes $n = 1$, $n = 2$, $n = 3$ Correct answer	1 1
12.15 (b)	$\frac{n+2}{2n+3}$	Numerator correct Denominator correct	1 1
12.16 (a)	$\frac{\sqrt{3}}{3},1,\sqrt{3}$	Substitutes $n = 1$, $n = 2$, $n = 3$ Two terms correct All terms correct	1 1 1
12.16 (b)	$5(\sqrt{2})^n$	5 $(\sqrt{2})^n$ or $2^{\frac{n}{2}}$	1 1

Question	Answer	Extra information	Marks
12.17	Rearrange one equation to match the format of the other, in order to compare them, term by term: $3y-4x=18$ $-y+10x=-32$ Multiply the second equation by 3 and then add the two equations: $3y-4x=18$ $-3y+30x=-96$ $26x=-78$ $x=-3$ Substitute $x=-3$ into either equation to find y. $3y-4(-3)=18$ $3y+12=18$ $3y=6$ $y=2$ Solution is $(-3, 2)$	Attempt to use a multiplier Add or subtract equations Solve for either x or y. Fully correct answer	1 1 1 1

Question	Answer	Extra information	Marks
12.18	Let p = cost of one pineapple, and b = cost of one banana Form two equations: $3p + 6b = 1710$ (1) $4p + 9b = 2405$ (2) Multiply (1) by 4 and (2) by 3: $12p + 24b = 6840$ $-\ 12p + 27b = 7215$ $-3b = -375$ $b = 125$ Cost of one banana = £1.25 $3p + 6 \times 125 = 1710$ $3p = 960$ $p = 320$ Cost of one pineapple = £3.20	Assign variables for the cost of one of each fruit Set up simultaneous equations Use multipliers to eliminate on variable Solve for either variable Substitute to solve for the other variable	1 1 1 1 1