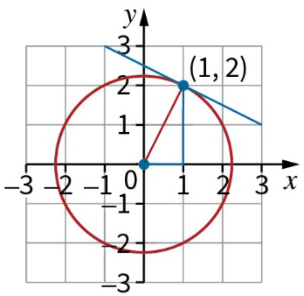
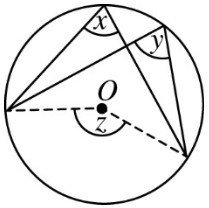
Oxford Revise | Edexcel GCSE Maths Higher | Answers

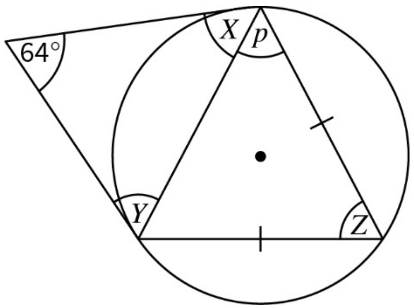
Chapter 26 Circle theorems and circle geometry

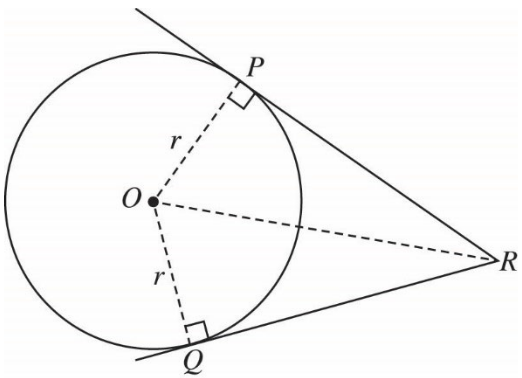
Question	Answer	Extra information	Marks
26.1	Substitute the coordinates $(1, 1)$ into the equation for the circle and show that it doesn't hold true: $1^2 + 1^2 \neq 1$	Substituting $(1, 1)$ into the equation Correct explanation	1 1
26.2 (a)	The radius is 4 and the centre is at $(0, 0)$, so the circle's equation is $x^2 + y^2 = 16$		1
26.2 (b)	Substitute $x = 2\sqrt{2}$ and $y = 2\sqrt{2}$ into the equation $x^2 + y^2 = 16$ and see if it is a true statement: $\left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2 = 8 + 8 = 16$ The statement holds true, so the point lies on the circle	Substituting $x = 2\sqrt{2}$ and $y = 2\sqrt{2}$ into the equation Showing that the statement holds true	1 1
26.2 (c)	$y = -4$		1

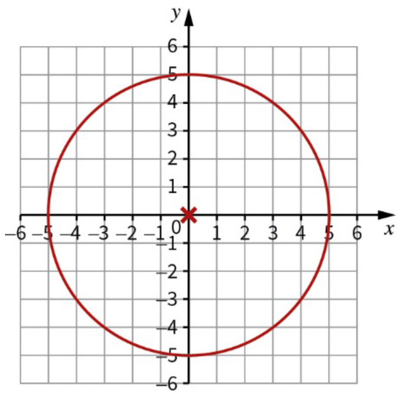
Question	Answer	Extra information	Marks
26.3	 Gradient of the radius from origin to $(1, 2) = \frac{2}{1} = 2$ The tangent is perpendicular to the radius, so its gradient $= -\frac{1}{2}$ Equation of tangent line is of the form $y = -\frac{1}{2}x + c$ Use the point $(1, 2)$ to find the value of c: $2 = -\frac{1}{2} \times 1 + c$ $c = 2.5$ Thus, the equation of the tangent is $y = 2.5 - 0.5x$	Attempt to calculate gradient of the radius Negative reciprocal for tangent's gradient Attempt to substitute into $y = mx + c$ Attempting to solve to find c. Correct answer	1 1 1 1 1

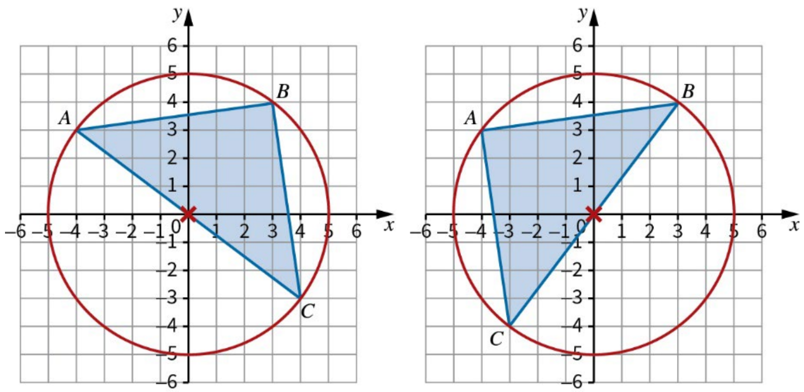
Question	Answer	Extra information	Marks
26.4	Show that all the angles are right angles and that all the sides are the same length. Points are $A\left(0,\sqrt{10}\right), B\left(\sqrt{10},0\right), C\left(0,-\sqrt{10}\right), D\left(-\sqrt{10},0\right)$ To show *ABC* is a right angle, find the gradient of *AB* and of *BC*: Gradient of *AB* $= \dfrac{\sqrt{10}}{-\sqrt{10}} = -1$ Gradient of *BC* $= \dfrac{\sqrt{10}}{\sqrt{10}} = 1$ These gradients are perpendicular, so *ABC* is a right angle. The same result can be found for the other three angles. To show that the sides have the same length, use Pythagoras to find that: *AB* = *BC* = *CD* = *DA* = $\sqrt{10+10} = \sqrt{20}$ Thus, *ABCD* is a square.	Finding the coordinates of *A*, *B*, *C* and *D* Finding the gradient of any of *AB*, *BC*, *CD* or *DA* Finding the length of any of *AB*, *BC*, *CD* or *DA* Full proof (which includes all sides same length, all angles $90°$) and conclusion.	1 1 1 1
26.5 (a)	$x = 90°$ (the angle in a semicircle is a right angle)	Correct answer Correct theorem stated	1 1
26.5 (b)	$x = 72°$ (angles in the same segment are equal)	Correct answer Correct theorem stated	1 1
26.5 (c)	$x = 80°$ (opposite angles in a cyclic quadrilateral sum to $180°$)	Correct answer Correct theorem stated	1 1
26.6 (a)	$x = 78°$ (angle at the centre is twice the angle at the circumference)	Correct answer Correct theorem stated	1 1

Question	Answer	Extra information	Marks
26.6 (b)	 You need to prove that $x = y$ First, draw the radii (see dotted lines). Now, you know that the angle at the centre is twice the angle at the circumference. Applying the theorem to x and to y, you have $z = 2x$ and $z = 2y$ so $2x = 2y$ and $x = y$, as required.	Diagram showing the correct theorem to be proved Drawing the radii Applying 'angle at the centre is twice the angle at the circumference' (must be stated clearly) Clearly deducing that $x = y$.	1 1 1 1
26.7	Angle *ADG* $= 90°$ (The angle in a semicircle is a right angle.) Angle *CED* $= 49°$ (Angles in a triangle sum to $180°$.) Angle *ACB* $= 41°$ and angle *FEG* $= 49°$ (Vertically opposite angles are equal.) Angle *EFG* $= 112°$ (Angles in a triangle sum to $180°$.) Angle *GAB* $= 180 - 112 = 68°$ (Opposite angles in a cyclic quadrilateral sum to $180°$.) $z = 68 - 21 = 47°$	*ADG* $= 90°$ *ACB* $= 41°$ or CED $= 49°$ Correct circle theorem used and stated $z = 47°$ Full geometric reasons given.	1 1 1 1 1
26.8 (a)	$x = 90°$; the angle between the tangent and radius is a right angle.	Correct answer Correct theorem stated	1 1

Question	Answer	Extra information	Marks
26.8 (b)	$x = 10$ cm; two tangents to a circle from the same point are equal in length.	Correct answer Correct theorem stated	1 1
26.8 (c)	$x = 81°$; the angle between the chord and tangent is equal to the angle in the alternate segment.	Correct answer Correct theorem stated	1 1
26.9	 Angle X = angle $Y = (180° – 64°) \div 2 = 58°$ (Tangents to a circle from a point are equal in length, and base angles of an isosceles triangle are equal and angles in a triangle sum to 180°.) Angle Z = angle X (= angle Y) = 58° (The angle between the chord and tangent is equal to the angle in the alternate segment.) $p = (180 – 58) \div 2 = 61°$ (Base angles of an isosceles triangle are equal and angles in a triangle sum to 180°.)	3 marks for $p = 61°$ (can be shown on the diagram) or 1 mark for angle X (or angle $Y = 58°$) (can be shown on the diagram) 1 mark for angle $Z = 58°$ (can be shown on the diagram) 1 mark for $p = 61°$; 1 mark for fully correct reasons stated throughout.	1 1 1 1

Question	Answer	Extra information	Marks
26.10	 You need to prove that length *PR* equals length QR. Let $OP = OQ = r$ By Pythagoras: $PR = \sqrt{OR^2 - r^2}$ Also, by Pythagoras: $QR = \sqrt{OR^2 - r^2}$ Thus $PR = QR$	Diagram showing the correct theorem to be proved and for drawing the radii, as well as line joining the centre to the exterior point Stating that angle between radius and tangent is a right angle Applying Pythagoras' theorem Fully correct and justified conclusion.	1 1 1 1
26.11 (a)	Use Pythagoras to show $(-4)^2 + 3^3 = 16 + 9 = 25$		1

Question	Answer	Extra information	Marks
26.11 (b)	 Circle with radius 5 Axis intercepts at $(5, 0)$, $(0, 5)$, $(–5, 0)$ and $(0, –5)$	Circle with centre at $(0, 0)$ and radius 5 Axis intercepts	1 1

Question	Answer	Extra information	Marks
26.11 (c)	 The angle is a semicircle is a right angle, so either *AC* or *BC* is a diameter. *AC* and *BC* would each pass through the origin. By symmetry, the possible coordinates of *C* are $(4, -3)$ and $(-3, -4)$	Stating the correct circle theorem Correct coordinates for both possibilities	1 1

Question	Answer	Extra information	Marks
26.12	Rearranging, $y=-x-6$ Substitute this value for *y* into the equation for the circle: $x^2+(-x-6)^2=18$ $x^2+x^2+12x+36=18$ $2x^2+12x+18=0$ $x^2+6x+9=0$ $(x+3)^2=0$ $x=-3$ The line meets the circle at just the one point, where $x=-3$ so it must be a tangent.	Rearranges and substitutes into the circle equation Expand and simplifies to form a quadratic Correct method to solve the quadratic Arrives at just one solution and makes appropriate conclusion	1 1 1 1
26.13	$OAB=OCB=90°$ Using the triangle *OAB*, $OA=\dfrac{8}{\tan 50°}=6.7127...$ Area of kite = $2\times\left(\dfrac{1}{2}\times 8\times 6.7127...\right)=53.7023...$ Area of sector = $2\pi\times 6.7127...\times\dfrac{100}{360}=11.7160...$ Shaded area = $53.7023\ldots - 11.7160\ldots = 42.0$ cm^2 (3 sf)	$OAB = 90°$ or $OCB = 90°$ Correct method to find radius of sector $(= 6.7127\ldots)$ Correct method for area of kite Correct method area of sector Correct answer to 3 significant figures.	1 1 1 1
26.14	Angle *VWX* = 84° (Alternate Segment Theorem) Angle *OXV* = 6° (Tangent/Radius) Angle *OVX* = 6° (Base angles of Isosceles Triangle) Using triangle *VXW*, Angle *OVW* = $180 - (6 + 6 + 39 + 84) = 45^\circ$	Angle *VWX* = $84°$ Angle *OXV* = $6°$ or Angle *OVX* = $6°$ Correct final answer	1 1 1

Question	Answer	Extra information	Marks
26.15	$\frac{6}{x+2}=\frac{9x+3}{6}$ $36=9x^2+18x+3x+6$ $9x^2+21x-30=0$ $3x^2+7x-10=0$ $(3x+10)(x-1)=0$ $x=-\frac{10}{3},\ x=1$ Disregard $x=-\frac{10}{3}$, because it is less than 0. Therefore $x=1$, and the first three terms are 3, 6, 12 Each term is twice the previous term; thus, the fifth term will be $12\times2\times2=48$	$\frac{6}{x+2}=\frac{9x+3}{6}$ $9x^2+21x-30=0$ $x=-\frac{10}{3},\ x=1$ Use $x=1$ to find first 3 terms of 3, 6 and 12 Finding the fifth term	1 1 1 1 1

Question	Answer	Extra information	Marks
26.16	*AB* $= 8.8$, *BC* $= 7.8$ and *AC* $= 5.6$ Let $a = BC, b = AC, c = AB$ Use the cosine rule to find the angle in question: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$ $= \cos^{-1}\left(\frac{8.8^2 + 5.6^2 - 7.8^2}{2 \times 8.8 \times 5.6}\right)$ $= 60.88217...$ To the nearest degree, the angle *A* $= 60.9°$	Identify that the problem requires the cosine rule, and attempt to use it $\cos^{-1}$(expression) attempted Fully correct, to 1 dp	1 1 1