Oxford Revise | Edexcel GCSE Maths Higher | Answers

Chapter 25 Trigonometry in 3D, sine and cosine rules

Question	Answer	Extra information	Marks
25.1	$12^2 + 3^2 + h^2 = 13^2$ $144 + 9 + h^2 = 169$ $h = 16$ $h = 4$ cm	$12^2 + 3^2 + h^2 = 13^2$ Correctly rearranged Correct answer	1 1 1
25.2	Same method as for a 2 cm × 2 cm × 1 cm cuboid Length of PQ = $\sqrt{2^2 + 2^2 + 1^2} = 3$ cm	1 mark for $\sqrt{2^2 + 2^2 + 1^2}$ Correct answer	1 1
25.3	Let the centre of the base be O and let the midpoint of AB be M. Using triangle AOM: $AO = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = 1$ cm Using triangle AEO: $h = \sqrt{\left(\sqrt{3}\right)^2 - 1^2} = \sqrt{2}$ cm	$AO = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}$ $h = \sqrt{\left(\sqrt{3}\right)^2 - 1^2}$ Correct answer	1 1 1

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25.4	Let *M* be the midpoint of *WX*. Using triangle *MOX*: $OX = \dfrac{1.5}{\cos 30°} = \sqrt{3}$ (Angle $OXM = 30°$ since OX bisects angle VXW which is $60°$ because VXW is an equilateral triangle.) Using triangle YOX : $\cos YXO = \dfrac{\sqrt{3}}{3}$ $\Rightarrow YXO = 54.735... = 55°$ to the nearest degree	Attempt to use $90°$, $60°$, $30°$ triangle $OX = \dfrac{1.5}{\cos 30°} = \sqrt{3}$ $\cos YXO = \dfrac{\sqrt{3}}{3}$ Correct answer to the nearest degree	1 1 1 1
25.5	$OP = \dfrac{12}{\tan 38°} = 15.539...$ cm $ST = \dfrac{7}{10} \times 20 = 14$ cm $PT = \sqrt{12^2 + 14^2} = 18.439...$ cm $\tan OTP = \dfrac{15.539...}{18.439...} = 0.832...$ $OTP = 39.793...$ $OTP = 39.8°$ to 1 decimal place	Correct trig. ratio used to find OP Uses ratio correctly to find ST Pythagoras used to find PT Correct trig. ratio used to find OTP Correct answer to 1 decimal place	1 1 1 1 1
25.6	$OC = \sqrt{2^2 + 3^2} = \sqrt{13}$ cm $\tan FOC = \dfrac{5}{\sqrt{13}}$ $FOC = 54.204...$ $= 54.2°$, to 1 decimal place	Pythagoras used to find OC Correct trig. ratio used to find FOC Correct answer to 1 decimal place	1 1 1

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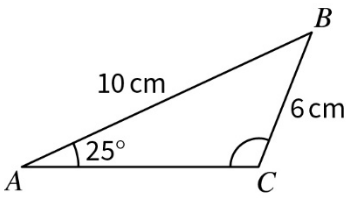
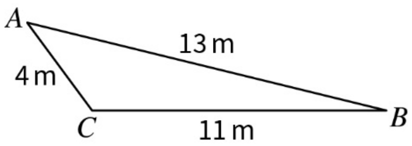
25.7	Using the cosine rule: $x^2 = 14^2 + 11^2 - 2\times14\times14\times\cos42° = 88.111...$ $x = 9.4$ cm, to 1 dp	$x^2 = 14^2 + 11^2 - 2\times14\times14\times\cos42°$ $88.111...$ Correct answer to 1 decimal place	1 1 1
25.7	Using the sine rule: $\frac{x}{\sin34°} = \frac{2.7}{\sin21°}$ $x = \frac{2.7\times\sin34°}{\sin21°} = 4.213... = 4.2$ cm, to 1 dp	$\frac{x}{\sin34°} = \frac{2.7}{\sin21°}$ $x = \frac{2.7\times\sin34°}{\sin21°}$ 4.2 cm, to 1 dp	1 1 1
25.8	Using the cosine rule: $\cos x = \frac{10.8^2 + 16.3^2 - 8.2^2}{2\times10.8\times16.3} = 0.894...$ $x = 26.499... = 26.5°$, to 1 dp	$\cos x = \frac{10.8^2 + 16.3^2 - 8.2^2}{2\times10.8\times16.3}$ $0.894...$ Correct answer to 1 dp	1 1 1
25.8	Using the sine rule: $\frac{\sin x}{13} = \frac{\sin67}{19}$ $\sin x = \frac{13\times\sin67}{19} = 0.6298...$ $x = 39.036... = 39.0°$, to 1 dp	$\frac{\sin x}{13} = \frac{\sin67°}{19}$ $\sin x = \frac{13\times\sin67°}{19}$ $39.0°$, to 1 dp	1 1 15

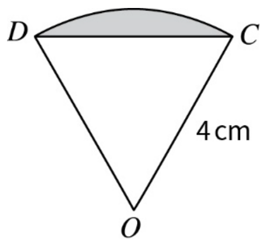
Question	Answer	Extra information	Marks
25.9	 Using the sine rule: $\frac{\sin ACB}{10} = \frac{\sin 25°}{6}$ $\sin ACB = \frac{10 \times \sin 25°}{6} = 0.704...$ $ACB = 44.778...$ ACB is obtuse, so $ACB = 180 - 44.778... = 135.2°$, 1 dp	$\frac{\sin ACB}{10} = \frac{\sin 25°}{6}$ $\sin ACB = \frac{10 \times \sin 25°}{6} = 0.704...$ $ACB = 44.778...$ Subtract $: 180 - 44.778...$ to obtain correct answer	1 1 1 1

Question	Answer	Extra information	Marks
25.10	Triangle ABD Using the sine rule: $\frac{BD}{\sin 70°} = \frac{16}{\sin 37°}$ $BD = \frac{\sin 70° \times 16}{\sin 37°} = 24.982...$ Triangle BCD Using the cosine rule: $BC^2 = 18^2 + BD^2 - 2 \times 18 \times BD \times \cos 43°$ $= 290.377...$ $BC =$ 17.0 cm, to 1 dp	$\frac{BD}{\sin 70°} = \frac{16}{\sin 37°}$ $BD = \frac{\sin 70° \times 16}{\sin 37°}$ $BC^2 = 18^2 + BD^2 - 2 \times 18 \times BD \times \cos 43°$ $290.377...$ Correct final answer	1 1 1 1 1
25.11	Area = $\frac{1}{2} \times 6 \times 9 \times \sin 32° = 14.31$ cm	Use of the sine formula to find the area Correct answer	1 1
25.11	Area = $\frac{1}{2} \times 10.21 \times 13.64 \times \sin 80° = 68.57$ cm	Use of the sine formula to find the area Correct answer	1 1

Question	Answer	Extra information	Marks
25.12	 Sketch the triangle. Using the cosine rule: $\cos CAB = \frac{4^2 + 13^2 - 11^2}{2 \times 4 \times 13} = \frac{8}{13}$ $CAB = 52.0201...°$ Area of triangle: $\frac{1}{2} \times 4° 13 \times \sin 52.0201$ $= 20.5$ m^2, to 3 sf	Correct use of cosine rule 52.0201… Correct use of area of triangle formula Correct answer, to 3 sf	1 1 1 1
25.13	Area of one face = $\frac{1}{2} \times 8 \times 8 \sin 60° = 16\sqrt{3}$ Surface area = $4 \times 16\sqrt{3} = 64\sqrt{3}$ cm^2	$60°$ Attempt to use $\frac{1}{2} ab \sin C$ $64\sqrt{3}$ Answer with correct units	1 1 1 1

Question	Answer	Extra information	Marks
25.14	Work out the areas of triangles *ABC* and *ACD* separately and then sum. $AC = 25\sin 32° = 13.2479...$ $BC = 25\cos 32° = 21.2012...$ Area of $ABC = \frac{1}{2} \times AC \times BC = 140.4365...$ You need to work out one of the angles in *ACD*. Using the cosine rule: $AC^2 = 36^2 + 26^2 - 2 \times 36 \times 26 \times \cos ADC$ $ADC = \cos^{-1}\left(\frac{36^2 + 26^2 - 13.2479...^2}{2 \times 36 \times 26}\right)$ Area of $ACD = \frac{1}{2} \times AD \times DC \times \sin ADC$ $= \frac{1}{2} \times 36 \times 26 \times \sin 16.3288...°$ $= 131.5779...$ Total area = 140.4365 + 131.4365 = 272.0 units^2 to 1 dp	Base or height of ABC Correct area of ABC Attempt to use the cosine rule Use of $\frac{1}{2}ab\sin C$ Correct rounded answer	1 1 1 1 1

Question	Answer	Extra information	Marks
25.15	Let h be the height of the cylinder, in cm. The volume of the prism in terms of h = $\frac{1}{4}\pi\times\left(\frac{11}{2}\right)^2\times h=\frac{121\pi h}{16}$ The length of the prism: $=\sqrt{14^2-4^2}=6\sqrt{5}$ The base of the prism is equilateral, so its angles are $60°$. Area of the base of the prism: $=\frac{1}{2}\times4\times4\times\sin60°=4\sqrt{3}$ Volume of prism = $4\sqrt{3}\times6\sqrt{5}=24\sqrt{15}$ Equating the two expressions for volume: $h=24\sqrt{15}\times\frac{16}{121\pi}=3.9123...$ The height of the container is 3.91 cm, to 3 sf	Using volume of a cylinder Using Pythagoras to find length of prism Area of triangle multiplied by length to work out prism volume Equating two expressions for volume and rearranging Correct rounded answer	1 1 1 1 1
25.16 (a)	Equilateral triangle, so angle $DOC=60°$ Area of triangle $OAB=\frac{1}{2}\times8\times8\times\sin60°=16\sqrt{3}$ Radius of circle $=8\div2=4$ Area of sector = $\frac{60}{360}\times\pi\times4^2=\frac{8\pi}{3}$ cm^2 Shaded area = $\left(16\sqrt{3}-\frac{8\pi}{3}\right)$ cm^2	$60°$ $\frac{1}{2}\times8\times8\times\sin60°$ $\frac{60}{360}\times\pi\times4^2$ Correct final answer or equivalent expression	1 1 1 1

Question	Answer	Extra information	Marks
25.16 (b)	 Sector area from part (a) = $\frac{8\pi}{3}$ cm^2 Triangle area = $\frac{1}{2}\times 4\times 4\times \sin 60°$ $= 4\sqrt{3}$ cm^2 Segment area = $\frac{8\pi}{3} - 4\sqrt{3} = 1.4493...$ Segment area = 1.4 cm^2, to 1 dp	Triangle area formula Subtracting from sector area Final answer, correct to 1 dp	1 1 1
25.17	$\frac{1}{2}\times 12\times AB\times \sin 39° = 80 AB = 21.1868$ $...BC^2 = 12^2 + AB^2 - 2\times 12\times AB\times \cos 39°$ $BC = 14.0611$ $...\frac{\sin 46°}{14.0611} = \frac{\sin CBD}{15} CBD = 50.1°$, to 1 dp	Use of area formula to find AB Use of cosine rule to find BC Use of sine rule to find CBD Correct answer Correct to 3 sf	1 1 1 1 1

Question	Answer	Extra information	Marks
25.18 (a)	The second digit must be either 1, 3, 5, 7 or 9, so that is 5 possibilities The first digit can be any digit except 0, so that makes 9 possibilities $9 \times 5 = 45$ possible different numbers	Identifying the number of possibilities for each digit. Multiplying them together	1 1
25.18 (b)	There are 90 possible numbers (10 – 99) and of these, there are 11 with the same digits (11, 22, 33, …. 99) Thus, the probability of one of these numbers being written is $\frac{11}{90}$	Identifying that there are 90 two-digit numbers overall Correct answer, either as a fraction or a decimal or a percentage	1 1
25.19	The three reflected vertices are: *A′*(2, 0), *B′*(0, –2) and *C′*(3, –4)	1 mark for two correct vertices 1 mark for all three	1 1