

Oxford Revise | Edexcel GCSE Maths Higher | Answers

Chapter 21 Pythagoras and 2D trigonometry

Question	Answer	Extra information	Marks
21.1	The only triangle that satisfies $a^2 + b^2 = c^2$ is the one with side lengths of 7 cm, 24 cm and 25 cm	Checking $a^2 + b^2 = c^2$ Correct answer	1 1
21.2	From the diagram, $(AB)^2 = 5^2 + 3^2 = 34$ AB = 5.83 to 3 sf	Identifying at least one of 3 and 5 as sides of a right-angled triangle Squaring and adding Taking the square root of your answer Correct answer to 3 sf	1 1 1



Question	Answer	Extra information	Marks
21.3	20 cm h $f = 20^2 - 5^2 = 375$ h = 19.364 = 19.4 cm to 1 dp	Splitting into two right-angled triangles with side 5 and squaring and subtracting; Taking the square root of your answer Correct answer to 1 dp	1 1 1
21.4	$(3h)^2 = 90^2 - 60^2 = 4500$ 3h = 67.082 h = 22.360 = 22.4 cm to 1 dp	Squaring and subtracting Taking the square root of your answer Dividing by 3 Correct answer to 1 dp	1 1 1 1
21.5 (a)	Volume = $\frac{1}{3}\pi \times 10^2 \times 24 = 800\pi \mathrm{cm}^3$	$\frac{1}{3}\pi \times 10^2 \times 24$ Correct answer	1
21.5 (b)	Slant height = $\sqrt{10^2 + 24^2} = 26$ Curved surface area = $\pi \times 10 \times 26 = 260\pi$ Base area = $\pi \times 10^2 = 100\pi$ Total surface area = $360\pi \text{ cm}^2$	Attempt to use Pythagoras to find slant height $\pi \times 10 \times$ slant height Base area Correct answer	1 1 1 1



Question	Answer	Extra information	Marks
21.6	$\sqrt{\frac{\sin 47^{\circ} \cos 21^{\circ}}{\cos 21^{\circ} - \sin 47^{\circ}}} = 1.84 \ (3 \text{ sf})$	Clear attempt to use calculator and write down at least 3 decimal places Correct answer, to 3 sf	1
21.7 (a)	$\sin 67^\circ = \frac{14}{x}$ $x = \frac{14}{\sin 67^\circ} = 15.2 \text{ cm}$	Correct ratio for sine Correct answer	1 1
21.7 (b)	$\cos 35^\circ = \frac{x}{16}$ x = 16 cos 35° = 13.1 cm	Correct ratio for cosine Correct answer	1 1
21.8	The trapezium can be split into a rectangle 24 cm by h cm and a right- angled triangle of height h Base of triangle = $40 - 24 = 16$ cm $h = 16 \tan 50^\circ = 19.1$ cm to 3 sf	40 - 24 (= 16) 16 tan 50 Correct answer to 3 sf.	1 1 1
21.9 (a)	y_1 360° 180 0 1 80 360 x	Correct cosine graph between 0° and 360° Fully correct graph for the whole range	1 1
21.9 (b)	(180, -1)		1



Question	Answer	Extra information	Marks
21.10	Using the left triangle in the diagram, the common side = $6.4 \div \tan 52^\circ = 5.0002$ cm Using the right triangle in the diagram, $b = 5.0002 \div \tan 31^\circ = 8.3$ cm to 1 dp	tan 52° (= 5.0002) Dividing answer by tan 31° Correct answer to 1 dp	1 1 1
21.11 (a)	$\sin x = \frac{4}{7} x = \sin^{-1}\left(\frac{4}{7}\right) = 34.8^{\circ}$	$\sin x = \frac{4}{7}$ Answer correct to 1 dp	1
21.11 (b)	$\tan x = \frac{10}{11}$ $x = \tan^{-1} \left(\frac{10}{11} \right) = 42.3^{\circ}$	$\tan x = \frac{10}{11}$ Answer correct to 1 dp	1 1
21.11 (c)	$\cos x = \frac{13}{20}$ $x = \cos^1\left(\frac{13}{20}\right) = 49.5^\circ$	$\cos x = \frac{13}{20}$ Answer correct to 1 dp	1



Question	Answer	Extra information	Marks
21.12	P R R $Cos PQR = \frac{6}{8}$ $\angle PQR = cos^{-1} \left(\frac{6}{8}\right) = 41.4^{\circ}$	$12 \div 2$ $\cos PQR = \frac{6}{8}$ Answer correct to 1 dp	1 1 1
21.13	$7 \text{ cm} \int_{A}^{C} \frac{16 \text{ cm}}{B}$ $\cos BCA = \frac{7}{16}$ $BCA = \cos^{-1}\left(\frac{7}{16}\right) = 64.1^{\circ}$	Correctly sketched and labelled $\cos BCA = \frac{7}{16}$ Answer correct to 1 dp	1 1 1



Question	Answer	Extra information	Marks
21.14 (a)	$\begin{array}{c} y \\ 1 \\ 0 \\ -1 \end{array}$	Correct section of graph of $\sin x$ between 0° and 360° Fully correct graph	1 1
21.14 (b)	$\theta = 135^\circ, 405^\circ$	135 405	1 1
21.15 (a)	$x = 4\sin 30^\circ = 4 \times 0.5 = 2$ cm	$4 \sin 30^{\circ}$ $\sin 30^{\circ} = 0.5$ Correct final answer	1 1 1
21.15 (b)	$x = 1 \times \tan 45^\circ = 1 \times 1 = 1 \text{ cm}$	$1 \times \tan 45^{\circ}$ tan 45° = 1 Correct final answer	1 1 1
21.15 (c)	$x = \frac{6}{\cos 60^\circ} = 12 \text{ cm}$	$\frac{6}{\cos 60^{\circ}}$ $\cos 60^{\circ} = 0.5$ Correct final answer	1 1 1



Question	Answer	Extra information	Marks
21.16 (a)	0° 30° 45° 60° 90° sin 0 $\frac{1}{2}$ $\frac{1}{\sqrt{2}}$ $\frac{\sqrt{3}}{2}$ 0 cos 1 $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{2}$ $\frac{1}{2}$ 0 tan 0 $\sqrt{3}$ 1 $\frac{\sqrt{3}}{2}$ undefined	1 mark for each correct answer; deduct 1 mark for each incorrect answer	4
21.16 (b)	$\sin 90^\circ = 1 \qquad \cos 45^\circ = \frac{\sqrt{2}}{2}$ $\tan 30^\circ = \frac{\sqrt{3}}{3} \qquad \tan 60^\circ = \sqrt{3}$	1 mark for each correct answer	4
21.17 (a)	$\cos x = \frac{1 + \sqrt{2}}{2 + \sqrt{2}} = \frac{1 + \sqrt{2}}{2 + \sqrt{2}} = \frac{1}{\sqrt{2}}$ $x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^{\circ}$	$\cos x = \frac{1 + \sqrt{2}}{2 + \sqrt{2}}$ $\cos x = \frac{1}{\sqrt{2}}$ Correct answer	1 1 1

OXFORD REVISE

Question	Answer	Extra information	Marks
21.17 (b)	$\tan 60 = \sqrt{3}$ $\sqrt{3} = \frac{x\sqrt{6}}{\sqrt{3}}$		1
	$2 - \sqrt{3}$ $x\sqrt{6} = 2\sqrt{3} - 3$ $2\sqrt{2} - 3$	$\tan 60^\circ = \sqrt{3}$ $\tan 60^\circ = \frac{x\sqrt{6}}{2-\sqrt{3}}$	1
	$x = \frac{2\sqrt{5-5}}{\sqrt{6}}$ Note that other forms are possible, depending on if the denominator is again rationalised.	Correct answer	1
21.18 (a)	$-180^{\circ} 1$ $180^{\circ} 360^{\circ} x$	Correct section of graph of tan x between 0° and 360° Fully correct graph.	1 1
21.18 (b)	$\tan 30^\circ = \frac{\sqrt{3}}{3}$		1
21.18 (c) (i)	$\tan 210^\circ = \tan 30^\circ = \frac{\sqrt{3}}{3}$		1
21.18 (c) (ii)	$\tan(-30)^{\circ} = -\tan 30^{\circ} = -\frac{\sqrt{3}}{3}$		1



Question	Answer	Extra information	Marks
21.19	y 2 1 90° 270° x 1 2	Horizontal translation of $y = \sin x$ Correct answer (translating 90 left).	1 1
21.20	$\begin{array}{c} y \\ 2 \\ -1 \\ -1 \\ -1 \\ 2 \\ -1 \\ -1 \\ -1 \\$	Reflecting $y = \cos x$ Correct answer (reflecting in the x-axis).	1 1
21.21	$\sin 45^\circ = \frac{\sqrt{2}}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{\sqrt{3}}{3}$ $(\sin 45^\circ)^2 = \frac{1}{2} \qquad (\cos 30^\circ)^2 = \frac{3}{4} \qquad (\tan 30^\circ)^2 = \frac{1}{3}$ Multiply through by 12 to give the ratio of 6 : 9 : 4 Thus $a = 6, b = 9, c = 4$ and so Tyrik is wrong.	Finding the three trig values Squaring the values Creating a ratio with integers	1 1 1
21.22	Base of right-angled triangle = 6 cm $EF^2 = 6^2 + 8^2$ EF = 10	6 cm stated Pythagoras correctly used Correct answer	1 1 1



Question	Answer	Extra information	Marks
21.23	$AE^{2} = 10.7^{2} - 6.2^{2}$ $AE = 8.7206$ $BE = 17.3 - 8.7206 = 8.5793$ $BF = 8.5793 7.6 = 0.9793$ $\tan ABC = \frac{62}{0.9793}$ $ABC = 81.0238$ $ABC = 81.0^{\circ} (1 \text{ dp})$	Pythagoras used correctly to find <i>AE</i> Process to find <i>BF</i> Correct use of tan <i>ABC</i> 81.0238 Correct answer, to 1 dp	1 1 1 1
21.24 (a)	$\tan ACB = \frac{5}{4}$ $ACB = \tan^{1}\left(\frac{5}{4}\right) = 51.3^{\circ}$	$\tan ACB = \frac{5}{4}$ Correct answer, to 1 dp	1 1
21.24 (b)	If the adjacent side increases, then the ratio of $\frac{\text{opposite}}{\text{adjacent}}$ will decrease, so the tangent of <i>ACB</i> will also decrease	Decrease, with acceptable reasoning	1



Question	Answer	Extra information	Marks
21.25	x = 702702702 1000x = 702.702702 999x = 702 $x = \frac{702}{999}$ $= \frac{26}{37}$	Use of $1000x$ Subtract to get $999x$ Correct final answer	1 1 1
20.26	$V = 60.75\pi = \frac{1}{2} \times \frac{4}{3}\pi r^{3}$ $60.75 = \frac{2}{3}r^{3}$ $91.125 = r^{3}$ $r = \sqrt[3]{91.125}$ = 4.5 The surface area is made up of two parts: the circular base, and the hemispheric shell: $SA = \pi r^{2} + \frac{1}{2} \times 4\pi r^{2} = 3\pi r^{2}$ $= 3\pi \times 4.25^{2} = 60.75\pi \text{ cm}^{2}$	Use the formula for the volume and take half of that Solve for the radius Use the radius to calculate the surface area Fully correct surface area Notice that the volume and surface area are numerically equal. (Will this always be true for any radius?)	1 1 1 1