Oxford Revise | Edexcel GCSE Maths Higher | Answers

Chapter 15 Algebraic fractions, rearranging formulae with algebraic fractions, proof, functions and composite functions

Question	Answer	Extra information	Marks
15.1 (a)	$(x-1)(x-2)(x-3)=(x^2-3x+2)(x-3)=x^3-3x^2+2x-3x^2+9x-6$ $=x^3-6x^2+11x-6$	x^2-3x+2 or x^2-5x+6 Any three correct terms in the final answer Fully correct	1 1 1
15.1 (b)	$n^4-6n^3+11n^2-6n$ $=n(n-1)(n-2)(n-3)$ from part (a) If n is even, then this is a multiple of 2 If n is odd, then $n-1$ is even, so again the expression contains a term that is a multiple of 2, and therefore the expression is always even for all $n>3$	Factorising Considering the cases when n is odd and even	1 1
15.2 (a)	$\frac{4x^2-12x}{2x}=\frac{2x(2x-6)}{2x}=2x-6=2(x-3)$	Factorising the numerator Correct answer	1 1
15.2 (b)	$\frac{x^2-x-2}{x^2-6x+8}=\frac{(x-2)(x+1)}{(x-2)(x-4)}=\frac{x+1}{x-4}$	Factorising the numerator Factorising the denominator Correct answer	1 1 1
15.3 (a)	$\frac{3x-4}{6x^2+7x-20}=\frac{3x-4}{(3x-4)(2x+5)}=\frac{1}{2x+5}$	Attempting to factorise the denominator Correct factorisation Correct answer	1 1 1

Question	Answer	Extra information	Marks
15.3 (b)	$\frac{3x-4}{6x^2+7x-20}=1 \Rightarrow \frac{1}{2x+5}=1$ $2x+5=1$ $2x=-4$ $x=-2$	Rearranging to $2x+5=1$ Correct answer	1 1
15.4	$\frac{1}{x-3}=\frac{x}{x+5}$ $x+5=x^2-3x$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5$ or $x=-1$	Cross multiplying Rearranging to form a quadratic Correct answer	1 1 1
15.5	$\frac{2x-8}{3x-15}\times\frac{x-5}{x-3}=\frac{2x-8}{3(x-5)}\times\frac{x-5}{x-3}$ $=\frac{2x-8}{3x-9}$ Also acceptable is further factorising: $=\frac{2(x-4)}{3(x-3)}$	Factorising Multiplying numerator & denominators Correct answer	1 1 1
15.6	$\frac{x}{x^2+2x-35}\div\frac{6x^3}{3x+21}=\frac{x}{x^2+2x-35}\times\frac{3x+21}{6x^3}$ $=\frac{x}{(x+7)(x-5)}\times\frac{3(x+7)}{6x^3}=\frac{1}{2x^2(x-5)}$	Factorising where possible Invert and multiply Multiplying numerators and denominators Correct answer	1 1 1 1

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15.7	The given ratio can be written as: $\frac{x^2}{x-1}=\frac{4}{1}$ Therefore, $x^2=4x-4$ $x^2-4x+4=0$ $(x-2)^2=0$ $x=2$	Setting up the ratio as a fraction equation Rearranging to form a quadratic Method to solve the quadratic Correct answer	1 1 1 1
15.8 (a)	$\frac{4}{x+1}+\frac{3}{x+2}=\frac{4(x+2)}{(x+1)(x+2)}+\frac{3(x+1)}{(x+1)(x+2)}$ $=\frac{4x+8+3x+3}{(x+1)(x+2)}$ $=\frac{7x+11}{(x+1)(x+2)}$	Finding the common denominator Correct numerator (unsimplified) Correct answer	1 1 1
15.8 (b)	$\frac{x}{x-2}-\frac{x-2}{x+3}=\frac{x(x+3)}{(x-2)(x+3)}-\frac{(x-2)(x-2)}{(x-2)(x+3)}$ $=\frac{x^2+3x-[x^2-4x+4]}{(x-2)(x+3)}$ $=\frac{7x-4}{(x-2)(x+3)}$	Finding the common denominator Correct numerator (unsimplified) Correct answer	1 1 1

Question	Answer	Extra information	Marks
15.9	Perimeter of equilateral triangle = $3\times\frac{3x-1}{5}=\frac{9x-3}{5}$ Perimeter of isosceles triangle = $\frac{x-1}{5}+2\times\frac{8x+1}{10}=\frac{x-1}{5}+\frac{8x+1}{5}=\frac{9x}{5}$ The difference is $\frac{9x}{5}-\frac{9x-3}{5}=\frac{3}{5}$ cm	Finding the perimeter of one triangle Finding the perimeter of both triangles subtracting one perimeter from the other Correct answer	1 1 1 1
15.10 (a) (i)	$\text{f}(1)=1^2-2\times1=-1$	Substitution into the function Correct answer	1 1
15.10 (a) (ii)	$\text{f}(-5)=(-5)^2-2\times(-5)$ $=25+10=35$	Substitution into the function Correct answer	1 1
15.10 (b) (i)	$x^2-2x=15$ $x^2-2x-15=0$ $(x-5)(x+3)=0$ $x=5$ or $x=-3$	Writing the function equal to 15 Method to solve the quadratic Two correct answers	1 1 1
15.10 (b) (ii)	$x^2-2x=4x$ $x^2-6x=0$ $x(x-6)=0$ $x=0$ or $x=6$	Writing the function equal to $4x$ Method to solve the quadratic Two correct answers	1 1 1
15.11 (a)	$\text{f}(1)=3\times1-1=2$ $\text{g}(2)=\frac{1}{2}$	First substitution Second substitution Correct answer	1 1 1

Question	Answer	Extra information	Marks
15.11 (b)	$\mathrm{g}\left(\frac{1}{2}\right)=2$ $\mathrm{f}(2)=3\times 2-1=5$	First substitution Second substitution Correct answer	1 1 1
15.12 (a)	$y=\frac{x-1}{2}$ $2y=x-1$ $x=2y+1$ So, $\mathrm{f}^{-1}(x)=2x+1$	Starting to rearrange Making x the subject Correct answer	1 1 1
15.12 (b)	$y=x^2-4$ $x^2=y+4$ $x=\sqrt{y+4}$ So, $\mathrm{g}^{-1}(x)=\sqrt{x+4}$	Starting to rearrange Making x the subject Correct answer	1 1 1
15.12 (c)	$y=\frac{x}{2x+3}$ $2xy+3y=x$ $2xy-x=-3y$ $x(2y-1)=-3y$ $x=\frac{3y}{1-2y}$ So, $\mathrm{h}^{-1}(x)=\frac{3x}{1-2x}$	Starting to rearrange Collecting x terms together Making x the subject Correct answer	1 1 1 1

Question	Answer	Extra information	Marks
15.13	$(m+n)^2+(m-n)^2 = m^2+2mn+n^2+m^2-2mn+n^2$ $= 2m^2+2n^2$ $= 2(m^2+n^2)$	Correctly expanding each set of brackets Correctly simplifying, before the final factorisation Correct answer	1 1 1
15.14 (a)	Let the first integer be n The next four consecutive integers are thus $n+1$, $n+2$, $n+3$ and $n+4$ The sum of these is $n+n+1+n+2+n+3+n+4=5n+10$ $=5(n+2)$ which is a multiple of 5.	Correct (unsimplified) sum $5n+10$ Factorising or showing that the simplified sum is divisible by 5	1 1 1
15.14 (b)	Let the numbers be $2m$ and $2n$ $(2m)^2+(2n)^2=4m^2+4n^2=4(m^2+n^2)$ which is a multiple of 4.	Correct (unsimplified) sum $4m^2+4n^2$ Showing that the simplified sum is divisible by 4	1 1 1
15.15	$\frac{2x^2+3x}{9x^2-4}\times\frac{9x^2+42x+24}{x^2+4x}=\frac{x(2x+3)}{(3x-2)(3x+2)}\times\frac{3(3x+2)(x+4)}{x(x+4)}$ $=\frac{3(2x+3)}{3x-2}$	Fully factorise any expression correctly Fully factorises any 2 expressions correctly Cancels at least one linear term correctly in numerator and denominator Correct final answer	1 1 1 1

Question	Answer	Extra information	Marks
15.16	$\frac{2}{x-1}+\frac{3}{x-2}+\frac{3}{2x}$ $=\frac{2(x-2)2x+3(x-1)2x+3(x-1)(x-2)}{2x(x-1)(x-2)}$ $=\frac{4x^2-8x+6x^2-6x+3x^2-9x+6}{2x(x-1)(x-2)}=\frac{13x^2-23x+6}{2x(x-1)(x-2)}$	Common denominator of $2x(x-1)(x-2)$ Correct numerators Attempt to expand and simplify Correct answer	1 1 1 1
15.17 (a)	$\mathrm{g}(1)=2$ So, $\mathrm{f}(2)=\frac{1}{2}$	Finding g(1) Correct answer	1 1
15.17 (b)	$\mathrm{gf}(x)=g\left(\frac{1}{x}\right)=\frac{2}{\left(\frac{1}{x}\right)^2}=2x^2$	Attempt at $\mathrm{g}\left(\frac{1}{x}\right)$ Correct answer	1 1
15.18 (a)	$\mathrm{ff}(x)=\mathrm{f}(2x+1)$ $=2(2x+1)+1$ $=4x+3$	Attempt at $\mathrm{f}(2x+1)$ Correct answer	1 1

Question	Answer	Extra information	Marks
15.18 (b)	Let $y = 2x + 1$ Interchange x and y to find $f^{-1}(x) = \frac{x-1}{2}$ $fg(x) = f(x^2) = 2x^2 + 1$ $f^{-1}(x) = fg(x) \Rightarrow \frac{x-1}{2} = 2x^2 + 1$ $4x^2 + 2 = x - 1$ $4x^2 - x + 3 = 0$	x and y interchanged Rearrange for y $\frac{x-1}{2}$ Attempt at $f(x^2)$ $2x^2 + 1$ Equates answers and attempts to rearrange Final correct answer with working clearly shown	1 1 1 1 1 1
15.19	Represent the two odd numbers with $2n-1$ and $2n+1$ The sum of these is $2n-1+2n+1=4n$ Alternative expressions possible for the two odd numbers, eg. $2n+1$ and $2n+3$ will also give an expression where a factor of 4 is obvious.	$2n-1$ and $2n+1$ or other suitable representations of the two numbers Adding the two together Convincing conclusion	1 1 1
15.20	$27.2 \times 10^6 \times 1.75 \times 10^{-3}$ $= 47\,600$ $= 4.7 \times 10^4$		1
15.21	$-12xy^2 + 15xyz - 3xy + 21ax^3y^2 = 3xy(-4y + 5z - x + 7ax^2y)$	Correctly factorising out any two of the three common factors, 3, x or y All three factors factored out. Also accept -3 being factored out instead of just 3 so long as all signs are reversed inside the bracket	1 1