Oxford Revise | Edexcel GCSE Maths Higher | Answers

Chapter 12 Sequences

Question	Answer	Extra information	Marks
12.1 (a)	First 4 terms: $7, 12, 17, 22$ Term-to-term rule: Add 5 Seventh term: 37 Hundredth term: 502 Type of sequence: Arithmetic		1 1 1 1 1
12.1 (b)	First 4 terms: $10, 20, 40, 80$ Term-to-term rule: Multiply by 2 Seventh term: 640 Hundredth term: 6.3×10^{30} (to 2 sf) Type of sequence: Geometric		1 1 1 1 1
12.1 (c)	First 4 terms: $-1, 0, 3, 8$ Term-to-term rule: Add $1, 3, 5, \ldots$ (add $2n - 1$) Seventh term: 35 Hundredth term: 9800 Type of sequence: Quadratic		1 1 1 1 1
12.1 (d)	Term-to-term rule: Add previous 2 terms Seventh term: 13 Type of sequence: Fibonacci		1 1 1
12.2 (a)	$8n + 3 = 51$; $8n = 48$; $n = 48 \div 8 = 6$ The 6th term is 51	Writing the equation Correct answer.	1 1
12.2 (b)	$8n + 3 = 64$; $8n = 61$ 61 is not divisible by 8, so 64 is not in the sequence.	Writing the equation Correct answer.	1 1

Question	Answer	Extra information	Marks
12.2 (c)	$8n + 3 > 100$; $8n > 97$; $> 97 \div 8$ $(= 12.125)$ This is the 13th term. The 13th term is $8 \times 13 + 3 = 107$	Writing the inequality 13th term Correct answer	1 1 1
12.3 (a)	When $n = 4$, $n^2 - 30 = 4^2 - 30$ $= 16 - 30 = -14$	Substituting in 4 Correct answer	1 1
12.3 (b)	$n^2 - 30 = 114$, so $n^2 = 144$. Since 144 is a square number, and $n = 12$, this is in the sequence.	Writing the equation Correct answer.	1 1
12.4 (a) (i)	With a Fibonacci sequence, you add together the previous two terms. The sequence begins: $m, n, m + n, m + 2n, 2m + 3n, 3m + 5n, 5m + 8n, \dots$ The fourth term is $m + 2n$		1
12.4 (a) (ii)	The seventh term is $5m + 8n$	Finding the fifth and sixth terms Correct answer.	1 1
12.4 (b)	$m = 3$ The gap between the 1st and 3rd is: $(m + n) - m = n$ so $n = 5$ The 8th term is $8m + 13n = 8 \times 3 + 13 \times 5 = 89$	Method for finding the 8th term Correct answer	1 1

<table>
<tr><th>Question</th><th>Answer</th><th>Extra information</th><th>Marks</th></tr>
<tr><td>12.5</td><td>
<table>
<tr><th></th><th>Sequence</th><th>Term-to-term rule</th><th>nth term</th><th>Tenth term</th></tr>
<tr><td>a</td><td>17, 23, 29, 35</td><td>Add 6</td><td>$6n + 11$</td><td>71</td></tr>
<tr><td>b</td><td>−1, 2, 5, 8</td><td>Add 3</td><td>$3n - 4$</td><td>26</td></tr>
<tr><td>c</td><td>4, 1, −2, −5</td><td>Subtract 3</td><td>$-3n + 7$ or $7 - 3n$</td><td>−23</td></tr>
<tr><td>d</td><td>20, 15, 10, 5</td><td>Subtract 5</td><td>$-5n + 25$ or $25 - 5n$</td><td>−25</td></tr>
<tr><td>e</td><td>3, 3.5, 4, 4.5</td><td>Add 0.5</td><td>$0.5n + 2.5$</td><td>7.5</td></tr>
</table>
</td><td></td><td>15</td></tr>
<tr><td>12.6</td><td>The sequence begins 5, __ , 11, …
Since it is arithmetic, it increases by the same amount each time. In two jumps, it increases by 6, so the term-to-term rule is ‘add 3’ and the sequence is 5, 8, 11, …
This makes the nth term $3n + 2$</td><td>Identifying the sequence
nth term.
50th & 60th term</td><td>1
1
1</td></tr>
<tr><td>12.7</td><td>The sequence 12, 9, 6, 3, …
has nth term $15 - 3n$
The 50th term is $15 - 3 \times 50 = -135$
and the 60th term is $15 - 3 \times 60 = -165$
The sum of these terms is
$(-135) + (-165) = -300$</td><td>Finding the nth term
Finding the 50th and 60th terms
Correct answer.</td><td>1
1
1</td></tr>
<tr><td>12.8 (a)</td><td>The next term will be $\frac{13}{6}$</td><td></td><td>1</td></tr>
<tr><td>12.8 (b)</td><td>The nth term is given by $\frac{2n+1}{n}$</td><td></td><td>3</td></tr>
</table>

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Question	Answer	Extra information	Marks
12.8 (c)	$\frac{2\times6+1}{6}\times\frac{2\times9+1}{9}=\frac{13}{6}\times\frac{19}{9}$ $=\frac{247}{54}$		2
12.9 (a)	5, 8, 13, 20, 29	At most 1 error Extra mark if all correct	1 1
12.9 (b)	1, 12, 27, 46, 69	At most 1 error Extra mark if all correct	1 1
12.9 (c)	44, 61, 68, 65, 52	At most 1 error Extra mark if all correct	1 1
12.10 (a)	n^2		1
12.10 (b)	$n^2 - 1$		1
12.10 (c)	$3n^2$		1
12.11 (a)	5, 8, 13, 20, … The difference between the terms forms an arithmetic sequence: 3, 5, 7, 9, 11, … The next two terms are 29 and 40	1 for each correct term	2
12.11 (b)	–2, 5, 14, 25, … The difference between the terms forms an arithmetic sequence: 7, 9, 11, 13, 15, … The next terms are 38 and 53	1 for each correct term	2

<table>
<tr><th>Question</th><th>Answer</th><th>Extra information</th><th>Marks</th></tr>
<tr><td>12.12</td><td>$n^2 + 2n + 2 = 50 \Rightarrow n^2 + 2n - 48 = 0$
$\Rightarrow (n + 8)(n - 6) = 0$
So, the solutions are $n = -8$ or $n = 6$
Since n is a positive number, $n = 6$
So, the 6th term is 50</td><td>Writing the nth term equal to 50
Rearranging to 0 and attempting to solve the quadratic by factorising (or equivalent method of solution)
Correct answer</td><td>1
1

1</td></tr>
<tr><td>12.13 (a)</td><td>Sequence: 10, 12, 16, 22
1st difference: 2 4 6
2nd difference: 2 2
2nd difference is 2, so in the quadratic
$an^2 + bn + c$
$a = 2 \div 2 = 1$
<table><tr><th>n</th><th>1</th><th>2</th><th>3</th><th>4</th></tr><tr><td></td><td>10</td><td>12</td><td>16</td><td>22</td></tr><tr><td>n^2</td><td>1</td><td>4</td><td>9</td><td>16</td></tr><tr><td>Difference</td><td>9</td><td>8</td><td>7</td><td>6</td></tr></table>
This is a linear sequence with nth term:
$-n + 10$
The nth term of the sequence is $n^2 - n + 10$</td><td>Finding $a = 1$
Attempting to find the second two terms of the nth term
Correct answer.</td><td>1
1

1</td></tr>
</table>

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<table>
<tr><th>Question</th><th>Answer</th><th>Extra information</th><th>Marks</th></tr>
<tr>
<td>12.13 (b)</td>
<td>Sequence: -9, 2, 17, 36
1st difference: 11 15 19
2nd difference: 4 4
2nd difference is 4, so in the quadratic
$an^2 + bn + c$
$a = 4 \div 2 = 2$
<table>
<tr><th>n</th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td></td><td>-9</td><td>2</td><td>17</td><td>36</td></tr>
<tr><td>$2n^2$</td><td>2</td><td>8</td><td>18</td><td>32</td></tr>
<tr><td>Difference</td><td>-11</td><td>-6</td><td>-1</td><td>4</td></tr>
</table>
This is a linear sequence with nth term:
$5n - 16$
The nth term of the sequence is $2n^2 + 5n - 16$</td>
<td>Finding $a = 2$
Attempting to find the second two terms of the nth term
Correct answer.</td>
<td>1
1

1</td>
</tr>
<tr>
<td>12.14</td>
<td>Since n is a positive integer, n^2 will always be positive and $4n$ will always be positive,
so $n^2 + 4n + 6$ will always be positive.
Alternatively, $n^2 + 4n + 6 = (n + 2)^2 + 2$
Since something squared is always greater than or equal to 0, $(n + 2)^2 + 2$ will always be positive.</td>
<td></td>
<td>1</td>
</tr>
</table>

Question	Answer	Extra information	Marks
12.15	$n=4 \Rightarrow 16a+b=42$ $n=9 \Rightarrow 81a+b=237$ Subtract the first equation from the second: $65a=195$ $a=3$ Substitute this into either equation to get $b=-6$ So, the nth term is $3n^2-6$ 15th term will be $3\times15^2-6=669$	Method to find an equation in a and b. Finds a pair of simultaneous equations, and an attempt to eliminate b. $a=3$ and $b=-6$ Substitutes $n=15$ into formula Correct final answer	1 1 1 1 1
12.16	$\dfrac{4}{9+\sqrt{y}}=\dfrac{9-\sqrt{y}}{4}$ $(9+\sqrt{y})(9-\sqrt{y})=16$ $81-y=16$ $y=65$	Sets up correct equation Attempt to expand and solve for y Correct answer	1 1 1
12.17 (a)	$\frac{1}{4},\frac{2}{5},\frac{3}{6}$	Substitutes $n=1$, $n=2$, $n=3$ Correct answer	1 1
12.17 (b)	$\dfrac{n+2}{2n+3}$	Numerator correct Denominator correct	1 1
12.18 (a)	$\dfrac{\sqrt{3}}{3},1,\sqrt{3}$	Substitutes $n=1$, $n=2$, $n=3$ Two terms correct All terms correct	1 1 1
12.18 (b)	$5(\sqrt{2})^n$	5 $(\sqrt{2})^n$ or $2^{\frac{n}{2}}$	1 1

Question	Answer	Extra information	Marks
12.19	Rearrange one equation to match the format of the other, in order to compare them, term by term: $3y-4x=18$ $-y+10x=-32$ Multiply the second equation by 3 and then add the two equations: $3y-4x=18$ $\underline{-3y+30x=-96}$ $26x=-78$ $x=-3$ Substitute $x=-3$ into either equation to find y. $3y-4(-3)=18$ $3y+12=18$ $3y=6$ $y=2$ Solution is $(-3, 2)$	Attempt to use a multiplier Add or subtract equations Solve for either x or y. Fully correct answer	1 1 1 1

Question	Answer	Extra information	Marks
12.20	Let p = cost of one pineapple, and b = cost of one banana Form two equations: $3p + 6b = 1710 \quad (1)$ $4p + 9b = 2405 \quad (2)$ Multiply (1) by 4 and (2) by 3: $12p + 24b = 6840$ $-\ \underline{12p + 27b = 7215}$ $-3b = -375$ $b = 125$ Cost of one banana = £1.25 $3p + 6 \times 125 = 1710$ $3p = 960$ $p = 320$ Cost of one pineapple = £3.20	Assign variables for the cost of one of each fruit Set up simultaneous equations Use multipliers to eliminate on variable Solve for either variable Substitute to solve for the other variable	1 1 1 1 1