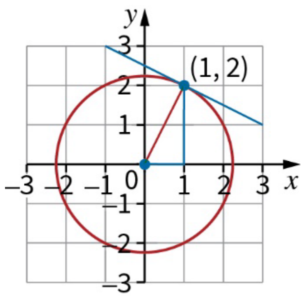
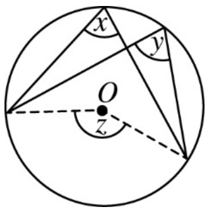
Oxford Revise | AQA GCSE Maths Higher | Answers

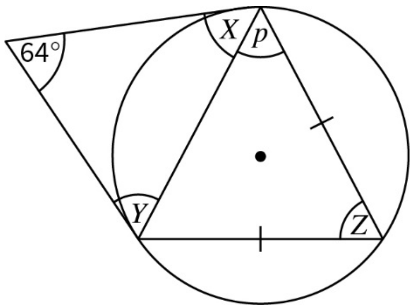
Chapter 26 Circle theorems and circle geometry

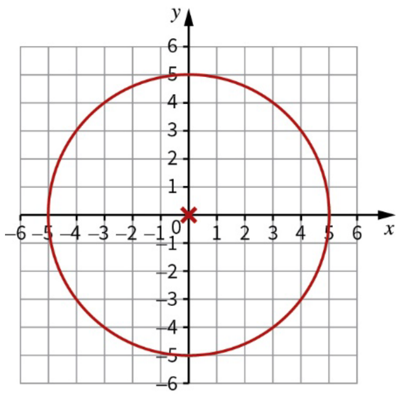
Question	Answer	Extra information	Marks
26.1	Substitute the coordinates $(1, 1)$ into the equation for the circle and show that it doesn't hold true: $1^2 + 1^2 \neq 1$	Substituting $(1, 1)$ into the equation Correct explanation	1 1
26.2 (a)	The radius is 4 and the centre is at $(0, 0)$, so the circle's equation is $x^2 + y^2 = 16$		1
26.2 (b)	Substitute $x = 2\sqrt{2}$ and $y = 2\sqrt{2}$ into the equation $x^2 + y^2 = 16$ and see if it is a true statement: $(2\sqrt{2})^2 + (2\sqrt{2})^2 = 8 + 8 = 16$ The statement holds true, so the point lies on the circle	Substituting $x = 2\sqrt{2}$ and $y = 2\sqrt{2}$ into the equation Showing that the statement holds true	1 1
26.2 (c)	$y = -4$		1

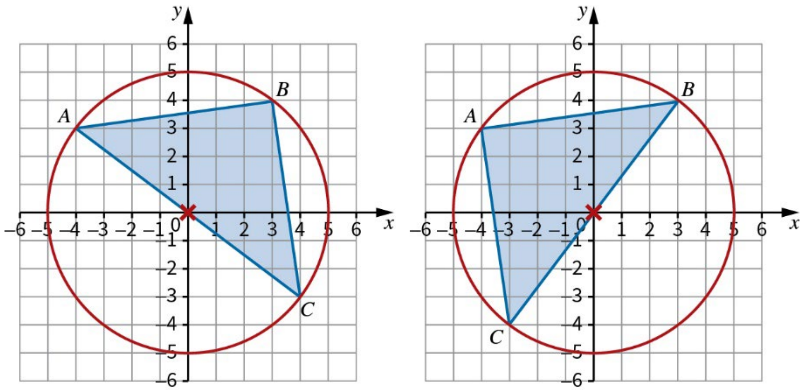
Question	Answer	Extra information	Marks
26.3	 Gradient of the radius from origin to $(1, 2) = \frac{2}{1} = 2$ The tangent is perpendicular to the radius, so its gradient $= -\frac{1}{2}$ Equation of tangent line is of the form $y = -\frac{1}{2}x + c$ Use the point $(1, 2)$ to find the value of c: $2 = -\frac{1}{2} \times 1 + c$ $c = 2.5$ Thus, the equation of the tangent is $y = 2.5 - 0.5x$	Attempt to calculate gradient of the radius Negative reciprocal for tangent's gradient Attempt to substitute into $y = mx + c$ Attempting to solve to find c. Correct answer	1 1 1 1 1

Question	Answer	Extra information	Marks
26.4	Show that all the angles are right angles and that all the sides are the same length. Points are $A\left(0,\sqrt{10}\right), B\left(\sqrt{10},0\right), C\left(0,-\sqrt{10}\right), D\left(-\sqrt{10},0\right)$ To show *ABC* is a right angle, find the gradient of *AB* and of *BC*: Gradient of $AB = \frac{\sqrt{10}}{-\sqrt{10}} = -1$ Gradient of $BC = \frac{\sqrt{10}}{\sqrt{10}} = 1$ These gradients are perpendicular, so *ABC* is a right angle. The same result can be found for the other three angles. To show that the sides have the same length, use Pythagoras to find that: $AB = BC = CD = DA = \sqrt{10+10} = \sqrt{20}$ Thus, *ABCD* is a square.	Finding the coordinates of *A*, *B*, *C* and *D* Finding the gradient of any of *AB*, *BC*, *CD* or *DA* Finding the length of any of *AB*, *BC*, *CD* or *DA* Full proof (which includes all sides same length, all angles $90°$) and conclusion.	1 1 1 1
26.5	$x = 72°$ (angles in the same segment are equal)	Correct answer Correct theorem stated	1 1
26.6 (a)	$x = 78°$ (angle at the centre is twice the angle at the circumference)	Correct answer Correct theorem stated	1 1

Question	Answer	Extra information	Marks
26.6 (b)	 You need to prove that $x = y$ First, draw the radii (see dotted lines). Now, you know that the angle at the centre is twice the angle at the circumference. Applying the theorem to x and to y, you have $z = 2x$ and $z = 2y$ so $2x = 2y$ and $x = y$, as required.	Diagram showing the correct theorem to be proved Drawing the radii Applying 'angle at the centre is twice the angle at the circumference' (must be stated clearly) Clearly deducing that $x = y$.	1 1 1 1
26.7	Angle *ADG* $= 90°$ (The angle in a semicircle is a right angle.) Angle *CED* $= 49°$ (Angles in a triangle sum to $180°$.) Angle *ACB* $= 41°$ and angle *FEG* $= 49°$ (Vertically opposite angles are equal.) Angle *EFG* $= 112°$ (Angles in a triangle sum to $180°$.) Angle *GAB* $= 180 - 112 = 68°$ (Opposite angles in a cyclic quadrilateral sum to $180°$.) $z = 68 - 21 = 47°$	*ADG* $= 90°$ *ACB* $= 41°$ or CED $= 49°$ Correct circle theorem used and stated $z = 47°$ Full geometric reasons given.	1 1 1 1 1
26.8	$x = 81°$; the angle between the chord and tangent is equal to the angle in the alternate segment.	Correct answer Correct theorem stated	1 1

Question	Answer	Extra information	Marks
26.9	 Angle X = angle $Y = (180° – 64°) \div 2 = 58°$ (Tangents to a circle from a point are equal in length, and base angles of an isosceles triangle are equal and angles in a triangle sum to $180°$.) Angle Z = angle X (= angle Y) $= 58°$ (The angle between the chord and tangent is equal to the angle in the alternate segment.) $p = (180 – 58) \div 2 = 61°$ (Base angles of an isosceles triangle are equal and angles in a triangle sum to $180°$.)	3 marks for $p = 61°$ (can be shown on the diagram) or 1 mark for angle X (or angle $Y = 58°$) (can be shown on the diagram) 1 mark for angle $Z = 58°$ (can be shown on the diagram) 1 mark for $p = 61°$; 1 mark for fully correct reasons stated throughout.	1 1 1 1
26.10 (a)	Use Pythagoras to show $(-4)^2 + 3^3 = 16 + 9 = 25$		1

Question	Answer	Extra information	Marks
26.10 (b)	 Circle with radius 5 Axis intercepts at $(5, 0)$, $(0, 5)$, $(–5, 0)$ and $(0, –5)$	Circle with centre at $(0, 0)$ and radius 5 Axis intercepts	1 1

Question	Answer	Extra information	Marks
26.10 (c)	 The angle is a semicircle is a right angle, so either *AC* or *BC* is a diameter. *AC* and *BC* would each pass through the origin. By symmetry, the possible coordinates of *C* are (4, –3) and (–3, –4)	Stating the correct circle theorem Correct coordinates for both possibilities	1 1

Question	Answer	Extra information	Marks
26.11	Rearranging, $y=-x-6$ Substitute this value for *y* into the equation for the circle: $x^2+(-x-6)^2=18$ $x^2+x^2+12x+36=18$ $2x^2+12x+18=0$ $x^2+6x+9=0$ $(x+3)^2=0$ $x=-3$ The line meets the circle at just the one point, where $x=-3$ so it must be a tangent.	Rearranges and substitutes into the circle equation Expand and simplifies to form a quadratic Correct method to solve the quadratic Arrives at just one solution and makes appropriate conclusion	1 1 1 1
26.12	$OAB=OCB=90°$ Using the triangle *OAB*, $OA=\frac{8}{\tan 50°}=6.7127...$ Area of kite = $2\times\left(\frac{1}{2}\times 8\times 6.7127...\right)=53.7023...$ Area of sector = $2\pi\times 6.7127...\times\frac{100}{360}=11.7160...$ Shaded area = $53.7023... - 11.7160... = 42.0$ cm^2 (3 sf)	$OAB = 90°$ or $OCB = 90°$ Correct method to find radius of sector $(= 6.7127...)$ Correct method for area of kite Correct method area of sector Correct answer to 3 significant figures.	1 1 1 1 1
26.13	Angle $VWX = 84°$ (Alternate Segment Theorem) Angle $OXV = 6°$ (Tangent/Radius) Angle $OVX = 6°$ (Base angles of Isosceles Triangle) Using triangle *VXW*, Angle $OVW = 180 - (6 + 6 + 39 + 84) = 45°$	Angle $VWX = 84°$ Angle $OXV = 6°$ or Angle $OVX = 6°$ Correct final answer	1 1 1

Question	Answer	Extra information	Marks
26.14	$\frac{6}{x+2}=\frac{9x+3}{6}$ $36=9x^2+18x+3x+6$ $9x^2+21x-30=0$ $3x^2+7x-10=0$ $(3x+10)(x-1)=0$ $x=-\frac{10}{3},\ x=1$ Disregard $x=-\frac{10}{3}$, because it is less than 0. Therefore $x=1$, and the first three terms are 3, 6, 12 Each term is twice the previous term; thus, the fifth term will be $12 \times 2 \times 2 = 48$	$\frac{6}{x+2}=\frac{9x+3}{6}$ $9x^2+21x-30=0$ $x=-\frac{10}{3},\ x=1$ Use $x=1$ to find first 3 terms of 3, 6 and 12 Finding the fifth term	1 1 1 1 1

Question	Answer	Extra information	Marks
26.15	*AB* $= 8.8$, *BC* $= 7.8$ and *AC* $= 5.6$ Let $a = BC, b = AC, c = AB$ Use the cosine rule to find the angle in question: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$ $= \cos^{-1}\left(\frac{8.8^2 + 5.6^2 - 7.8^2}{2 \times 8.8 \times 5.6}\right)$ $= 60.88217...$ To the nearest degree, the angle *A* $= 60.9°$	Identify that the problem requires the cosine rule, and attempt to use it $\cos^{-1}$(expression) attempted Fully correct, to 1 dp	1 1 1