

# Oxford Revise | AQA GCSE Maths Higher | Answers

## Chapter 15 Algebraic fractions, rearranging formulae with algebraic fractions, proof, functions and composite functions

Question	Answer	Extra information	Marks
15.1 (a)	$(x-1)(x-2)(x-3) = (x^2 - 3x + 2)(x-3)$ $= x^3 - 3x^2 + 2x - 3x^2 + 9x - 6$ $= x^3 - 6x^2 + 11x - 6$	$x^2 - 3x + 2$ or $x^2 - 5x + 6$ Any three correct terms in the final answer Fully correct	1 1 1
15.1 (b)	$n^4 - 6n^3 + 11n^2 - 6n$ $= n(n-1)(n-2)(n-3)$ from part (a) If $n$ is even, then this is a multiple of 2 If $n$ is odd, then $n-1$ is even, so again the expression contains a term that is a multiple of 2, and therefore the expression is always even for all $n > 3$	Factorising Considering the cases when $n$ is odd and even	1 1
15.2 (a)	$\frac{4x^2 - 12x}{2x} = \frac{2x(2x-6)}{2x} = 2x-6 = 2(x-3)$	Factorising the numerator Correct answer	1 1
15.2 (b)	$\frac{x^2 - x - 2}{x^2 - 6x + 8} = \frac{(x-2)(x+1)}{(x-2)(x-4)} = \frac{x+1}{x-4}$	Factorising the numerator Factorising the denominator Correct answer	1 1 1
15.3 (a)	$\frac{3x-4}{6x^2 + 7x - 20} = \frac{3x-4}{(3x-4)(2x+5)} = \frac{1}{2x+5}$	Attempting to factorise the denominator Correct factorisation Correct answer	1 1 1

Question	Answer	Extra information	Marks
15.3 (b)	$\frac{3x-4}{6x^2+7x-20} = 1 \Rightarrow \frac{1}{2x+5} = 1$ $2x+5=1$ $2x=-4$ $x=-2$	Rearranging to $2x+5=1$ Correct answer	1 1
15.4	$\frac{1}{x-3} = \frac{x}{x+5}$ $x+5 = x^2 - 3x$ $x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $x = 5 \text{ or } x = -1$	Cross multiplying Rearranging to form a quadratic Correct answer	1 1 1
15.5	$\frac{2x-8}{3x-15} \times \frac{x-5}{x-3} = \frac{2x-8}{3(x-5)} \times \frac{x-5}{x-3}$ $= \frac{2x-8}{3x-9}$ <p>Also acceptable is further factorising:</p> $= \frac{2(x-4)}{3(x-3)}$	Factorising Multiplying numerator & denominators Correct answer	1 1 1
15.6	$\frac{x}{x^2+2x-35} \div \frac{6x^3}{3x+21} = \frac{x}{x^2+2x-35} \times \frac{3x+21}{6x^3}$ $= \frac{x}{(x+7)(x-5)} \times \frac{3(x+7)}{6x^3} = \frac{1}{2x^2(x-5)}$	Factorising where possible Invert and multiply Multiplying numerators and denominators Correct answer	1 1 1 1

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15.7	<p>The given ratio can be written as:</p> $\frac{x^2}{x-1} = \frac{4}{1}$ <p>Therefore, <math>x^2 = 4x - 4</math></p> $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $x = 2$	<p>Setting up the ratio as a fraction equation</p> <p>Rearranging to form a quadratic</p> <p>Method to solve the quadratic</p> <p>Correct answer</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
15.8	$\frac{4}{x+1} + \frac{3}{x+2} = \frac{4(x+2)}{(x+1)(x+2)} + \frac{3(x+1)}{(x+1)(x+2)}$ $= \frac{4x+8+3x+3}{(x+1)(x+2)}$ $= \frac{7x+11}{(x+1)(x+2)}$	<p>Finding the common denominator</p> <p>Correct numerator (unsimplified)</p> <p>Correct answer</p>	<p>1</p> <p>1</p> <p>1</p>
15.9	$\frac{2}{x+4} - \frac{3}{x-2} = \frac{2(x-2)}{(x+4)(x-2)} - \frac{3(x+4)}{(x+4)(x-2)}$ $= \frac{2x-4-[3x+12]}{(x+4)(x-2)}$ $= \frac{-x-16}{(x+4)(x-2)}$	<p>Finding the common denominator</p> <p>Correct numerator (unsimplified)</p> <p>Correct answer</p>	<p>1</p> <p>1</p> <p>1</p>
15.10 (a) (i)	$f(1) = 1^2 - 2 \times 1 = -1$	<p>Substitution into the function</p> <p>Correct answer</p>	<p>1</p> <p>1</p>

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15.10 (a) (ii)	$f(-5) = (-5)^2 - 2 \times (-5)$ $= 25 + 10 = 35$	Substitution into the function Correct answer	1 1
15.10 (b) (i)	$x^2 - 2x = 15$ $x^2 - 2x - 15 = 0$ $(x - 5)(x + 3) = 0$ $x = 5$ or $x = -3$	Writing the function equal to 15 Method to solve the quadratic Two correct answers	1 1 1
15.10 (b) (ii)	$x^2 - 2x = 4x$ $x^2 - 6x = 0$ $x(x - 6) = 0$ $x = 0$ or $x = 6$	Writing the function equal to $4x$ Method to solve the quadratic Two correct answers	1 1 1
15.11 (a)	$f(1) = 3 \times 1 - 1 = 2$ $g(2) = \frac{1}{2}$	First substitution Second substitution Correct answer	1 1 1
15.11 (b)	$g\left(\frac{1}{2}\right) = 2$ $f(2) = 3 \times 2 - 1 = 5$	First substitution Second substitution Correct answer	1 1 1
15.12	$y = x^2 - 4$ $x^2 = y + 4$ $x = \sqrt{y + 4}$ So, $g^{-1}(x) = \sqrt{x + 4}$	Starting to rearrange Making $x$ the subject Correct answer	1 1 1

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15.13	$(m+n)^2 + (m-n)^2 = m^2 + 2mn + n^2 + m^2 - 2mn + n^2$ $= 2m^2 + 2n^2$ $= 2(m^2 + n^2)$	Correctly expanding each set of brackets Correctly simplifying, before the final factorisation Correct answer	1 1 1
15.14	Let the first integer be $n$ The next four consecutive integers are thus $n+1, n+2, n+3$ and $n+4$ The sum of these is $n+n+1+n+2+n+3+n+4 = 5n+10$ $= 5(n+2)$ which is a multiple of 5.	Correct (unsimplified) sum $5n+10$ Factorising or showing that the simplified sum is divisible by 5	1 1 1
15.15	$\frac{2x^2+3x}{9x^2-4} \times \frac{9x^2+42x+24}{x^2+4x} = \frac{x(2x+3)}{(3x-2)(3x+2)} \times \frac{3(3x+2)(x+4)}{x(x+4)}$ $= \frac{3(2x+3)}{3x-2}$	Fully factorise any expression correctly Fully factorises any 2 expressions correctly Cancels at least one linear term correctly in numerator and denominator Correct final answer	1 1 1 1

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15.16	$\frac{2}{x-1} + \frac{3}{x-2} + \frac{3}{2x}$ $= \frac{2(x-2)2x + 3(x-1)2x + 3(x-1)(x-2)}{2x(x-1)(x-2)}$ $= \frac{4x^2 - 8x + 6x^2 - 6x + 3x^2 - 9x + 6}{2x(x-1)(x-2)} = \frac{13x^2 - 23x + 6}{2x(x-1)(x-2)}$	Common denominator of $2x(x-1)(x-2)$ Correct numerators Attempt to expand and simplify Correct answer	1 1 1 1
15.17 (a)	$g(1) = 2$ So, $f(2) = \frac{1}{2}$	Finding $g(1)$ Correct answer	1 1
15.17 (b)	$gf(x) = g\left(\frac{1}{x}\right) = \frac{2}{\left(\frac{1}{x}\right)^2} = 2x^2$	Attempt at $g\left(\frac{1}{x}\right)$ Correct answer	1 1
15.18 (a)	$ff(x) = f(2x+1)$ $= 2(2x+1)+1$ $= 4x+3$	Attempt at $f(2x+1)$ Correct answer	1 1

Question	Answer	Extra information	Marks
15.18 (b)	<p>Let <math>y = 2x + 1</math></p> <p>Interchange <math>x</math> and <math>y</math> to find <math>f^{-1}(x) = \frac{x-1}{2}</math></p> <p><math>fg(x) = f(x^2) = 2x^2 + 1</math> <math>f^{-1}(x) = fg(x) \Rightarrow \frac{x-1}{2} = 2x^2 + 1</math></p> <p><math>4x^2 + 2 = x - 1</math></p> <p><math>4x^2 - x + 3 = 0</math></p>	<p><math>x</math> and <math>y</math> interchanged</p> <p>Rearrange for <math>y</math></p> $\frac{x-1}{2}$ <p>Attempt at <math>f(x^2)</math></p> $2x^2 + 1$ <p>Equates answers and attempts to rearrange</p> <p>Final correct answer with working clearly shown</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
15.19	<p>Represent the two odd numbers with <math>2n - 1</math> and <math>2n + 1</math></p> <p>The sum of these is <math>2n - 1 + 2n + 1 = 4n</math></p> <p>Alternative expressions possible for the two odd numbers, eg. <math>2n + 1</math> and <math>2n + 3</math> will also give an expression where a factor of 4 is obvious.</p>	<p><math>2n - 1</math> and <math>2n + 1</math> or other suitable representations of the two numbers</p> <p>Adding the two together</p> <p>Convincing conclusion</p>	<p>1</p> <p>1</p> <p>1</p>
15.20	$27.2 \times 10^6 \times 1.75 \times 10^{-3}$ $= 47\,600$ $= 4.7 \times 10^4$		1
15.21	$-12xy^2 + 15xyz - 3xy + 21ax^3y^2 = 3xy(-4y + 5z - x + 7ax^2y)$	<p>Correctly factorising out any two of the three common factors, 3, <math>x</math> or <math>y</math></p> <p>All three factors factored out.</p> <p>Also accept <math>-3</math> being factored out instead of just 3 so long as all signs are reversed inside the bracket</p>	<p>1</p> <p>1</p>

